Educate Cloud

JUNE 2017

QP and MA

HIGHER

QUESTION

The table shows a set of values for x and y.

x	1	2	3	4
y	9	$2\frac{1}{4}$	1	$\frac{9}{16}$

y is inversely proportional to the square of x.

(a) Find an equation for y in terms of x.

(b) Find the positive value of x when $y = 16$

MODEL ANSWER

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(a) Find an equation for y in terms of x.

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2}$

Sub $x = 3$ $y = 1$ $\quad 1 = \frac{k}{(3)^2}$

$1 = \frac{k}{9}$

$\times 9$ $\quad \times 9$

$9 = k$

$y = 9/x^2$

(2)

(b) Find the positive value of x when $y = 16$

Sub $y = 16$ in eq^n

$16 = \frac{9}{x^2} \Rightarrow x^2 = \frac{9}{16} \Rightarrow x = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

$\times \frac{x^2}{16}$

$3/4$

(2)

QUESTION

White shapes and black shapes are used in a game.
Some of the shapes are circles.
All the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is $3:7$

The ratio of the number of white circles to the number of white squares is $4:5$

The ratio of the number of black circles to the number of black squares is $2:5$

Work out what fraction of all the shapes are circles.

MODEL ANSWER

White : Black
3 : 7

White – C : S
4 : 5

Black – C : S
2 : 5

Assume there are 300 shapes in total:

White : $\frac{3}{10} \times 300 = 90$ (Total whites)

C : S
4 : 5 (×10)
40 : 50 (×10)
40 circles

Black : $\frac{7}{10} \times 300 = 210$ (Total Blacks)

C : S
2 : 5 (×30)
60 : 150 (×30)
60 circles

Total circles = 40 + 60 = 100

$\therefore \frac{100}{300} = \frac{1}{3}$ (Total circles → 100, Total shapes → 300)

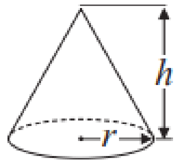
1/3

QUESTION

A cone has a volume of 98 cm^3.
The radius of the cone is 5.13 cm.

(a) Work out an estimate for the height of the cone.

Volume of cone = $\frac{1}{3}\pi r^2 h$



John uses a calculator to work out the height of the cone to 2 decimal places.

(b) Will your estimate be more than John's answer or less than John's answer?
Give reasons for your answer.

MODEL ANSWER

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$98 \approx 100$

$5.13 \approx 5$

$\pi \approx 3.14 \approx 3$

} approximated to 1 sf

$\therefore \quad 100 = \frac{1}{3}\pi(5)^2 h$

$100 = \frac{1}{3} \times 3 \times 25 \times h$

$100 = 25h$

$h = 4$

(b) Will your estimate be more than John's answer or less than John's answer?
Give reasons for your answer.

The v would be smaller for John and the π and r will be larger. ∴ When the numerator is small and the denominator is large, our estimate will be larger.

$\frac{\text{Small}}{\text{large}} = \text{Small}$. (for John)

(1)

(Total for Question 15 is 4 marks)

QUESTION

n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

MODEL ANSWER

Expanding; $n^2-2-[(n-2)(n-2)]$ Expand the brackets

$= n^2-2-[n^2-2n-2n+4]$ Simplify

$= n^2-2-[n^2-4n+4]$ Expand the square bracket.

$= n^2-2-n^2+4n-4$

$= 4n-6$

$= 2(2n-3)$ × To prove can be divisible by 2.

$4n-6$ is always even as it can be divided by 2.

QUESTION

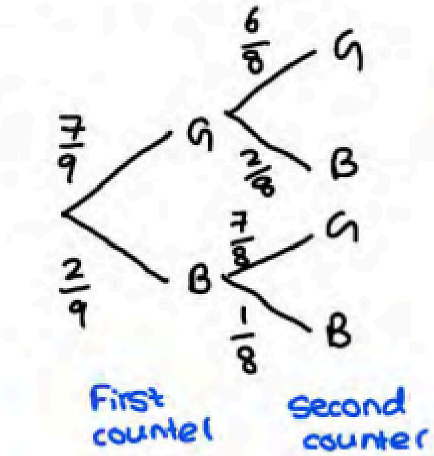
There are 9 counters in a bag.

7 of the counters are green.
2 of the counters are blue.

Ria takes at random two counters from the bag.

Work out the probability that Ria takes one counter of each colour.
You must show your working.

MODEL ANSWER



Draw a tree diagram to illustrate the probability distribution.

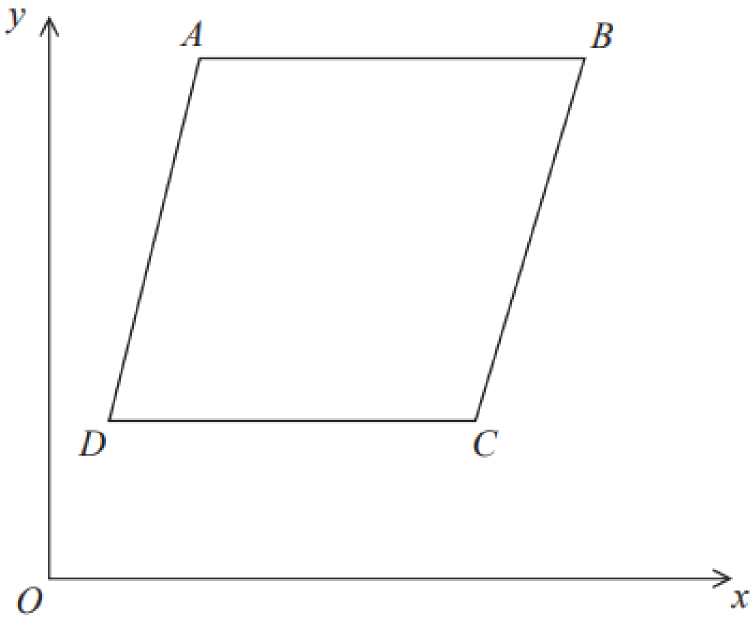
$$P(GB) + P(BG) = [P(G) \times P(B)] + [P(B) \times P(G)]$$

$$= \frac{7}{9} \times \frac{2}{8} + \frac{2}{9} \times \frac{7}{8}$$

$$= \frac{14}{72} + \frac{14}{72}$$

$$= \underline{\underline{\frac{28}{72}}}$$

QUESTION



ABCD is a rhombus.
The coordinates of *A* are $(5, 11)$
The equation of the diagonal *DB* is $y = \frac{1}{2}x + 6$

Find an equation of the diagonal *AC*.

MODEL ANSWER

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DB is perpendicular to AC, as it is a rhombus.

Grad of DB = $\frac{1}{2}$ ← from $y=\frac{1}{2}x+6$

Grad of AC = -2 ← negative reciprocal as its perpendicular

AC// $y=mx+c$

$y=-2x+c$

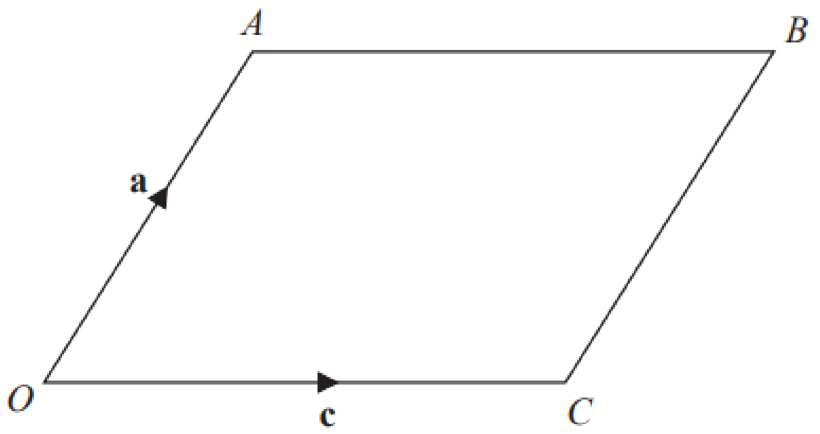
Sub (5,11) to eq^n ⇒ $11=-2(5)+c$) +10

$c=11+10$

$=21$

$y=-2x+21$

QUESTION



$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \dfrac{1}{2}\mathbf{a}$

find the value of k.

MODEL ANSWER

find the value of k.

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\underset{\sim}{a} + \underset{\sim}{c}$

$= \underset{\sim}{c} - \underset{\sim}{a}$

$\overrightarrow{XD} = \overrightarrow{XC} + \overrightarrow{CD}$

$= \frac{1}{2}\overrightarrow{AC} + \overrightarrow{CD}$

Substitute $\overrightarrow{XD}$ and $\overrightarrow{AC}$

$3\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a} = \frac{1}{2}[\underset{\sim}{c} - \underset{\sim}{a}] + \overrightarrow{CD}$

$3\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a} = \frac{1}{2}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a} + \overrightarrow{CD}$

Subject $\overrightarrow{CD}$

$\overrightarrow{CD} = 3\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a}$

$= 3\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{a}$

$= 2.5\underset{\sim}{c}$

OC : CD

compare ratios.

$k : 1$

$\underset{\sim}{c} : 2.5\underset{\sim}{c}$ $\quad \div 2.5\underset{\sim}{c}$

$\frac{1}{2.5} : 1$

$\therefore k = \frac{1}{2.5} = \frac{2}{5}$

$k = \frac{2}{5}$

(Total for Question 19 is 4 marks)

QUESTION

Solve algebraically the simultaneous equations

$$x^2 + y^2 = 25$$
$$y - 3x = 13$$

MODEL ANSWER

Subject y in (2)

$$y = 13 + 3x \quad \text{—(3)}$$

Subs. (3) into (1)

$$x^2 + (13+3x)^2 = 25$$

$$x^2 + (13+3x)(13+3x) = 25$$

$$x^2 + 169 + 39x + 39x + 9x^2 = 25$$

$$10x^2 + 78x + 169 - 25 = 0$$

$$\div 2 \quad 10x^2 + 78x + 144 = 0$$

$$5x^2 + 39x + 72 = 0$$

$$(5x+24)(x+3) = 0$$

$$\Rightarrow 5x + 24 = 0 \qquad \Rightarrow x + 3 = 0$$

$$x = -\frac{24}{5} \qquad x = -3$$

Sub x into (3)

$x = -\frac{24}{5}$

$$y = 13 + 3\left(-\frac{24}{5}\right)$$

$$= 13 - \frac{72}{5}$$

$$= -7/5$$

$x = -3$

$$y = 13 + 3(-3)$$

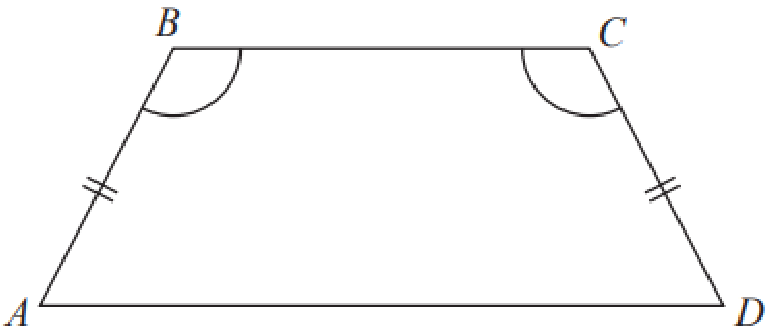
$$= 13 - 9$$

$$= 4$$

$x = -\frac{24}{5}$, $y = -7/5$; $x = -3$, $y = 4$

QUESTION

$ABCD$ is a quadrilateral.



$AB = CD$.
Angle ABC = angle BCD.

Prove that $AC = BD$.

MODEL ANSWER

$AB = CD$ (shown in diagram)

$B\hat{A}D = C\hat{D}A$

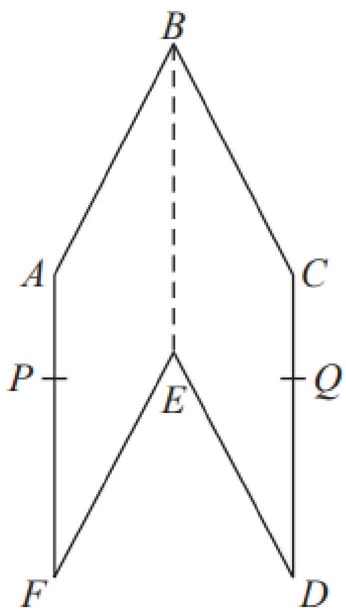
BC is a common side

Triangles are congruent under SAS.

$\therefore AC = BC$.

QUESTION

The diagram shows a hexagon *ABCDEF*.



ABEF and *CBED* are congruent parallelograms where $AB = BC = x$ cm.
P is the point on *AF* and *Q* is the point on *CD* such that $BP = BQ = 10$ cm.

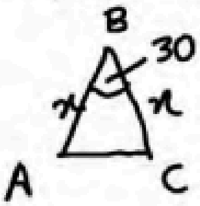
Given that angle $ABC = 30°$,

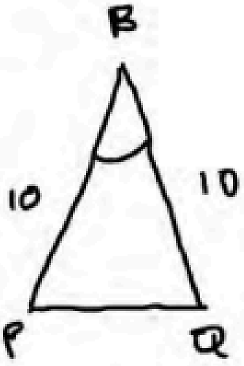
prove that $\cos PBQ = 1 - \frac{(2 - \sqrt{3})}{200}x^2$

MODEL ANSWER

$\cos 30 = \frac{\sqrt{3}}{2}$

$AC = PQ$





$$AC^2 = x^2 + x^2 - 2(x)(x)\cos 30$$
$$= 2x^2 - 2x^2\left(\frac{\sqrt{3}}{2}\right)$$
$$= x^2(2-\sqrt{3})$$

$$PQ^2 = 10^2 + 10^2 - 2(10)(10)\cos PBQ$$

$$x^2(2-\sqrt{3}) = 10^2 + 10^2 - 2(10)^2 \cos PBQ$$

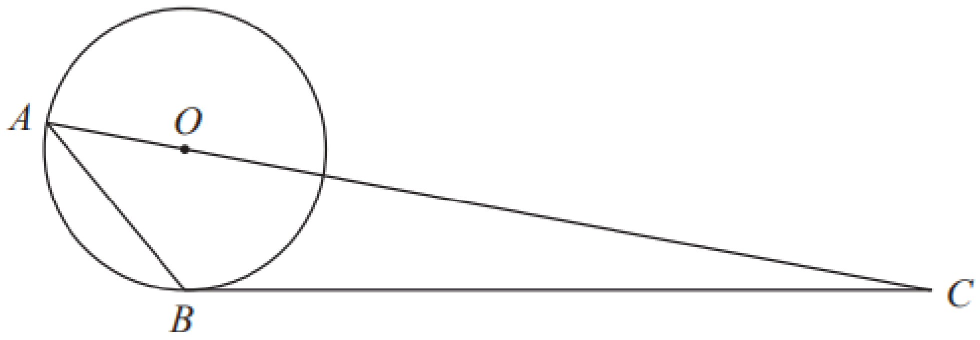
$$\cos PBQ = \frac{10^2 + 10^2 - (2-\sqrt{3})x^2}{2(10)^2} = \frac{200 - (2-\sqrt{3})x^2}{200}$$
$$= 1 - \frac{(2-\sqrt{3})x^2}{200} \quad //$$

JUNE 2018

QP and MA

HIGHER

QUESTION



A and B are points on a circle, centre O.

BC is a tangent to the circle.
AOC is a straight line.
Angle $ABO = x°$.

Find the size of angle ACB, in terms of x.
Give your answer in its simplest form.
Give reasons for each stage of your working.

MODEL ANSWER

angle OBC = 90° (radius through B is perpendicular to tangent at B)

triangle ABO is isosceles as AO = BO = radius of circle

angle BAO = angle ABO = x (base angles in isosceles triangle are equal)

triangle ABC: $x + x + 90° + A\hat{C}B = 180°$ (sum of internal angles of a triangle is 180°)

$A\hat{C}B = 180 - 90 - x - x = 90 - 2x$

angle ACB = $90 - 2x$

QUESTION

Prove that the square of an odd number is always 1 more than a multiple of 4

MODEL ANSWER

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odd number: $2n+1$ (where n is any integer)

$$(2n+1)^2 = (2n+1)(2n+1) = 4n^2 + 2n + 2n + 1$$
$$= 4n^2 + 4n + 1$$
$$= 4(n^2+n) + 1$$

$4(n^2+n)$ is a multiple of 4

$4(n^2+n)+1$ is 1 more than a multiple of 4

QUESTION

$\sqrt{5}(\sqrt{8} + \sqrt{18})$ can be written in the form $a\sqrt{10}$ where a is an integer.

Find the value of a.

MODEL ANSWER

$$\sqrt{5}(\sqrt{8}+\sqrt{18}) = (\sqrt{8}\times\sqrt{5})+(\sqrt{18}\times\sqrt{5})$$
$$= \sqrt{40}+\sqrt{90}$$

$$\sqrt{40} = \sqrt{4}\times\sqrt{10} = 2\sqrt{10}$$
$$\sqrt{90} = \sqrt{9}\times\sqrt{10} = 3\sqrt{10}$$

$$\sqrt{40}+\sqrt{90} = 2\sqrt{10}+3\sqrt{10} = 5\sqrt{10}$$

$$a = 5$$

QUESTION

y is inversely proportional to d^2
When $d = 10$, $y = 4$

d is directly proportional to x^2
When $x = 2$, $d = 24$

Find a formula for y in terms of x.
Give your answer in its simplest form.

MODEL ANSWER

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$y \propto \frac{1}{d^2} \rightarrow y = \frac{k}{d^2}$

$y = 4,\ d = 10$ substitute in values

$4 = \frac{k}{10^2} = \frac{k}{100}$ $\quad k = 4 \times 100 = 400$

$y = \frac{400}{d^2}$

$d \propto x^2 \rightarrow d = kx^2$

↑ constant of proportionality

$x = 2,\ d = 24$

$24 = k(2)^2 = 4k$ $\quad k = 24 \div 4 = 6$

$d = 6x^2$

cancel common factors

$y = \frac{400}{(6x^2)^2} = \frac{400}{36x^4} = \frac{\overset{100}{\cancel{400}}\ \div 4}{\underset{\div 4}{9\ \cancel{36}}x^4} = \frac{100}{9x^4}$

$y = \frac{100}{9x^4}$

QUESTION

(a) Factorise $a^2 - b^2$

(b) Hence, or otherwise, simplify fully $(x^2 + 4)^2 - (x^2 - 2)^2$

MODEL ANSWER

$a+b)(a-b)$ difference of 2 squares

$(a+b)(a-b)$

(1)

b) Hence, or otherwise, simplify fully $(x^2 + 4)^2 - (x^2 - 2)^2$

$$a^2 - b^2$$

$$(x^2+4)^2 - (x^2-2)^2$$

$$=\left[(x^2+4)+(x^2-2)\right]\left[(x^2+4)-(x^2-2)\right]$$

$= (2x^2+2) \times 6$ collect like terms

$= 6(2x^2+2)$ take out

$= 12(x^2+1)$

$12(x^2+1)$

(3)

QUESTION

There are only red counters, blue counters and purple counters in a bag.
The ratio of the number of red counters to the number of blue counters is 3 : 17

Sam takes at random a counter from the bag.
The probability that the counter is purple is 0.2

Work out the probability that Sam takes a red counter.

MODEL ANSWER

red : blue

3 : 17

$p(\text{purple}) = 0.2 = \frac{1}{5}$

$p(\text{not purple}) = 1 - \frac{1}{5} = \frac{4}{5}$

$p(\text{not purple}) = p(\text{red or blue}) = \frac{4}{5}$

$3 + 17 = 20$

$p(\text{a specific red or blue counter is selected}) = \frac{4}{5} \div 20 = \frac{4}{100} = 0.04$

$p(\text{red}) = 3 \times 0.04 = 0.12$

0.12

QUESTION

Simplify fully $\dfrac{3x^2 - 8x - 3}{2x^2 - 6x}$

MODEL ANSWER

$$\frac{3x^2 - 8x - 3}{2x^2 - 6x} = \frac{(3x+1)\cancel{(x-3)}}{2x\cancel{(x-3)}} = \frac{3x+1}{2x}$$

cancel common factors

factorise

$3x^2 - 8x - 3$

$-3 \times 3 = -9$

$-9 \times 1 = -9$ ✓

$-9 + 1 = -8$ ✓

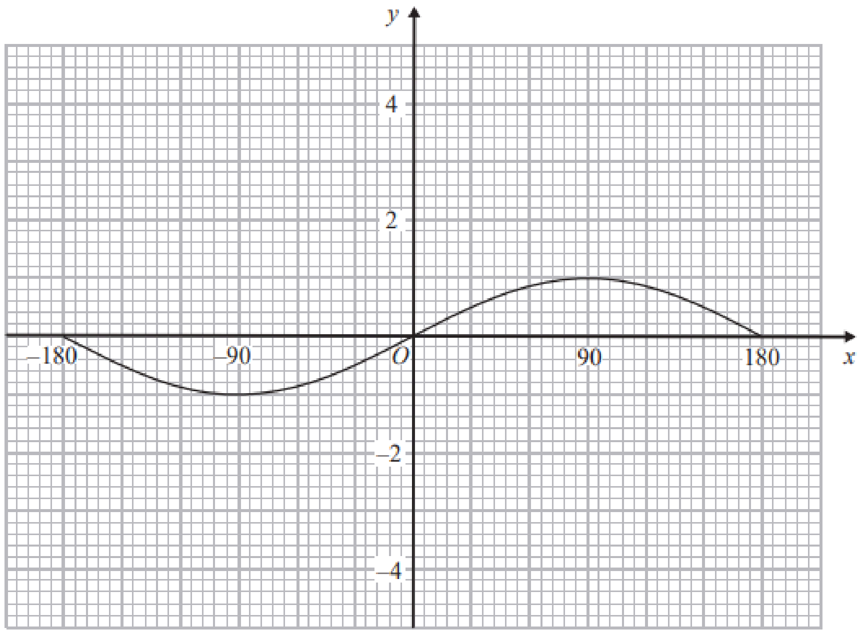
$3x^2 - 9x + x - 3$ ← split term in middle

$(3x^2 - 9x) + (x - 3)$

$3x(x-3) + (x-3) \rightarrow (3x+1)(x-3)$

QUESTION

Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

MODEL ANSWER

On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ 2 units ↓

QUESTION

The point P has coordinates $(3, 4)$
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $3x + 2y = 7$

Find an expression for b in terms of a.

MODEL ANSWER

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gradient of PQ = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{b-4}{a-3}$

Line perpendicular to PQ: $3x + 2y = 7$

$2y = -3x + 7$

$\rightarrow y = \frac{-3}{2}x + \frac{7}{2}$

$y = mx + c \qquad m = -\frac{3}{2}$

gradient of perpendicular lines

$m_{PQ} \times m = -1$

$m_{PQ} \times \frac{-3}{2} = -1$

$m_{PQ} = \frac{2}{3}$

$\frac{b-4}{a-3} = \frac{2}{3}$

cross multiply

$\frac{b-4}{a-3} = \frac{2}{3}$

$3(b-4) = 2(a-3)$

$3b - 12 = 2a - 6$ rearrange to make b the subject

$3b = 2a + 6$

$b = \frac{2}{3}a + 2$

QUESTION

n is an integer such that $3n + 2 \leqslant 14$ and $\dfrac{6n}{n^2 + 5} > 1$

Find all the possible values of n.

MODEL ANSWER

$3n+2 \leqslant 14$

$-2 \quad -2$

$3n \leqslant 12$

$\div 3 \quad \div 3$

$n \leqslant 4$

$$\frac{6n}{n^2+5} > 1$$

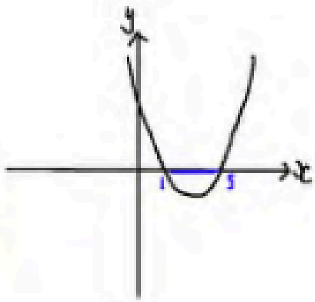
$6n > (n^2+5)$

$n^2 - 6n + 5 < 0$

$(n-5)(n-1) < 0$

$1 < n < 5$

sketch graph



$n = 2, 3, 4$

JUNE 2019

QP and MA

HIGHER

QUESTION

Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

MODEL ANSWER

If n is odd: $n^2 - n = (2n+1)^2 - (2n+1)$

$= 4n^2 + 4n + 1 - (2n+1)$

$= 4n^2 + 2n = \underline{\underline{2(2n^2+1)}}$ → multiple of 2 ∴ even.

(1)

If n is even: $n^2 - n = (2n)^2 - (2n)$ (1)

$= 4n^2 - 2n = \underline{\underline{2(2n^2-1)}}$ → multiple of 2 ∴ even.

$n^2 - n$ is even when n is odd and when n is even.

∴ $n^2 - n$ is never an odd number.

QUESTION

Find the exact value of $\tan 30° \times \sin 60°$
Give your answer in its simplest form.

MODEL ANSWER

$\tan 30 = \frac{1}{\sqrt{3}}$. $\sin 60 = \frac{\sqrt{3}}{2}$.

$\therefore \tan 30^\circ \times \sin 60^\circ = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

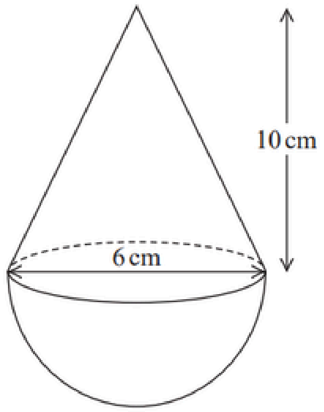
$= \frac{\sqrt{3}}{2\sqrt{3}}$ (1)

$= \boxed{\frac{1}{2}}$

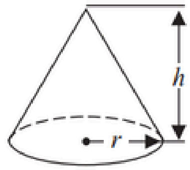
(1) $\frac{1}{2}$.

QUESTION

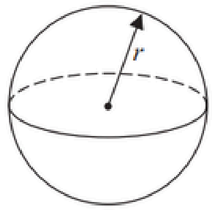
The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.



Volume of a cone $= \frac{1}{3}\pi r^2 h$



Volume of a sphere $= \frac{4}{3}\pi r^3$



The height of the cone is 10 cm.
The base of the cone has a diameter of 6 cm.
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

MODEL ANSWER

Volume of cone:

diameter = 6 cm ∴ radius $= \frac{6}{2} = 3$ cm. (1)

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3^2)(10) = \frac{9 \times 10 \times \pi}{3} = \frac{90}{3}\pi = \underline{\underline{30\pi}}$.

Volume of hemisphere:

Hemisphere $= \frac{1}{2}$(sphere) → Volume of hemisphere is half the volume of sphere.

$V_{hemisphere} = \left(\frac{4}{3}\pi r^3\right) \div 2 = \frac{4}{6}\pi r^3$. (1)

$\frac{4}{6}\pi r^3 = \frac{4}{6}\pi(3^3) = \frac{4 \times 27 \times \pi}{6} = \frac{108}{6}\pi = \underline{\underline{18\pi}}$.

Total volume $= 30\pi + 18\pi = \underline{\underline{48\pi \text{ cm}^3}}$ (1)

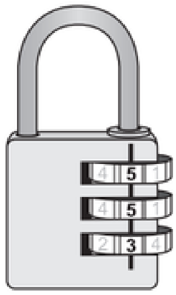
(1)

∴ k = 48.

k = 48

QUESTION

There are three dials on a combination lock.
Each dial can be set to one of the numbers 1, 2, 3, 4, 5
The three digit number 553 is one way the dials can be set, as shown in the diagram.



(a) Work out the number of different three digit numbers that can be set for the combination lock.

(b) How many of the possible three digit numbers have three different digits?

MODEL ANSWER

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DIGIT 1

1, 2, 3, 4, 5 ⟶ 5 POSSIBLE DIGITS

DIGIT 2

1, 2, 3, 4, 5 ⟶ 5 POSSIBLE DIGITS

DIGIT 3

1, 2, 3, 4, 5 ⟶ 5 POSSIBLE DIGITS

∴ number of three-digit numbers $= 5 \times 5 \times 5 = 125$ (1)

125 (1)

(2)

(b) How many of the possible three digit numbers have three different digits?

DIGIT 1

1, 2, 3, 4, 5 ⟶ 5 POSSIBLE DIGITS

DIGIT 2

2, 3, 4, 5 ⟶ 4 POSSIBLE DIGITS

DIGIT 3

3, 4, 5 ⟶ 3 POSSIBLE DIGITS

∴ number of three-digit numbers with three different digits $= 5 \times 4 \times 3 = 60$ (1)

60 (1)

(2)

QUESTION

Given that

$$x^2 : (3x + 5) = 1 : 2$$

find the possible values of x.

MODEL ANSWER

$$x^2 : (3x+5) = 1 : 2$$

$$\frac{x^2}{3x+5} = \frac{1}{2} \quad (1)$$

cross-multiply: $2x^2 = 3x + 5$

(1)

$\therefore 2x^2 - 3x - 5 = 0$

$(2x-5)(x+1) = 0$

(1)

$(x+1) = 0 \rightarrow x = -1$

$(2x-5) = 0 \rightarrow x = \frac{5}{2}$

(1)

$x = -1,\ x = \frac{5}{2}$

QUESTION

(a) Express $\sqrt{3} + \sqrt{12}$ in the form $a\sqrt{3}$ where a is an integer.

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

MODEL ANSWER

Educate Cloud

$$\sqrt{3} + \sqrt{12}$$
$$= \sqrt{3} + \sqrt{4 \times 3}$$
$$= \sqrt{3} + (\sqrt{4})(\sqrt{3})$$
$$= \sqrt{3} + 2\sqrt{3} \quad (1)$$
$$= \underline{\underline{3\sqrt{3}}}$$

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

$$\left(\frac{1}{\sqrt{3}}\right)^7 = \frac{(1)^7}{(\sqrt{3})^7}$$

$$(\sqrt{3})^7 = (\sqrt{3})^6 \times (\sqrt{3})^1$$
$$= (3^{\frac{1}{2}})^6 \times (\sqrt{3})^1$$
$$= 3^3 \times \sqrt{3} = 27\sqrt{3} \quad (1)$$

$$\therefore \frac{(1)^7}{(\sqrt{3})^7} = \frac{1}{27\sqrt{3}} \;\; (\times \sqrt{3}) \;\; = \frac{\sqrt{3}}{(27\sqrt{3})(\sqrt{3})} = \frac{\sqrt{3}}{27 \times 3} = \boxed{\frac{\sqrt{3}}{81}}$$

(1)

QUESTION

Given that $x^2 - 6x + 1 = (x - a)^2 - b$ for all values of x,

(i) find the value of a and the value of b.

(ii) Hence write down the coordinates of the turning point on the graph of $y = x^2 - 6x + 1$

MODEL ANSWER

Educate Cloud

$x^2 - 6x + 1$

$= (x-3)^2 - 3^2 + 1$ (1)

Because $(-6) \div 2 = -3$

$= (x-3)^2 - 9 + 1$

$= (x-3)^2 - 8$

If $y = (x+a)^2 + b$,

the turning point

$= (-a, b)$

$\therefore$ the turning point of $(x-3)^2 - 8$

$= (3, -8)$

(1) (3 , -8)

(1)

(Total for Question 19 is 3 marks)

QUESTION

h is inversely proportional to p
p is directly proportional to $\sqrt{t}$

Given that $h = 10$ and $t = 144$ when $p = 6$
find a formula for h in terms of t

MODEL ANSWER

$h = \frac{a}{p}$. $p = b\sqrt{t}$. (1)

Formula for h: $10 = \frac{a}{6}$, $a = 60$. (1) $h = \frac{60}{p}$.

Formula for p: $6 = b\sqrt{144}$, $6 = b(12)$, $b = \frac{6}{12} = \frac{1}{2}$. $p = \frac{1}{2}\sqrt{t}$. (1) $p = \frac{\sqrt{t}}{2}$.

Formula for h in terms of t:

$h = \frac{60}{p}$. $h = \frac{60}{\left(\frac{\sqrt{t}}{2}\right)}$.

$h = 60 \div \frac{\sqrt{t}}{2}$.

$h = 60 \times \frac{2}{\sqrt{t}}$.

$h = \frac{120}{\sqrt{t}}$

(1) $h = \frac{120}{\sqrt{t}}$

(Total for Question 20 is 4 marks)

QUESTION

The functions f and g are such that

$$f(x) = 3x - 1 \quad \text{and} \quad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

MODEL ANSWER

$f(x) = 3x - 1$

$y = 3x - 1$ (1)

$y + 1 = 3x$

$\div 3$ $\dfrac{y+1}{3} = x$ $\div 3$

$x = \dfrac{y+1}{3}$

$\therefore f^{-1}(x) = \dfrac{x+1}{3}$

$f^{-1}(x) =$ (1) $\dfrac{x+1}{3}$

(2)

Find $fg(x)$:

$fg(x) = f(g(x)) = f(x^2+4)$

$f(x^2+4) = 3(x^2+4) - 1 = 3x^2 + 12 - 1 = 3x^2 + 11$

$\rightarrow fg(x) = 3x^2 + 11$ (1)

Find $gf(x)$:

$gf(x) = g(f(x)) = g(3x-1)$

$g(3x-1) = (3x-1)^2 + 4 = (9x^2 - 6x + 1) + 4 = 9x^2 - 6x + 5$ (1)

$\rightarrow gf(x) = 9x^2 - 6x + 5$ (1)

$fg(x) = 2gf(x)$ (1)

$3x^2 + 11 = 2(9x^2 - 6x + 5)$

$3x^2 + 11 = 18x^2 - 12x + 10$

$0 = 15x^2 - 12x - 1$

(1) $\therefore 15x^2 - 12x - 1 = 0$

(5)

QUESTION

There are only r red counters and g green counters in a bag.

A counter is taken at random from the bag.
The probability that the counter is green is $\frac{3}{7}$
The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.
The probability that the counter is green is $\frac{6}{13}$

Find the number of red counters and the number of green counters that were in the bag originally.

MODEL ANSWER

	P(Green)	P(Red)
original	$\frac{3x}{7x}$	$\frac{4x}{7x}$
After more counters added.	$\frac{3x+3}{7x+5}$	$\frac{4x+2}{7x+5}$ (1)

$$\frac{3x+3}{7x+5} = \frac{6}{13} \quad (1)$$

$$13(3x+3) = 6(7x+5)$$

$$39x + 39 = 42x + 30 \quad (1)$$

$$39 = 3x + 30$$

$$9 = 3x$$

$$\therefore x = 3 \quad (1)$$

$\therefore$ number of red =

$4x = 4(3) = \boxed{12}$

number of green =

$= 3x = 3(3) = \boxed{9}$ (1)

red counters 12

green counters 9

JUNE 2020

QP and MA

HIGHER

QUESTION

Sally plays two games against Martin.
In each game, Sally could win, draw or lose.

In each game they play,

the probability that Sally will win against Martin is 0.3
the probability that Sally will draw against Martin is 0.1

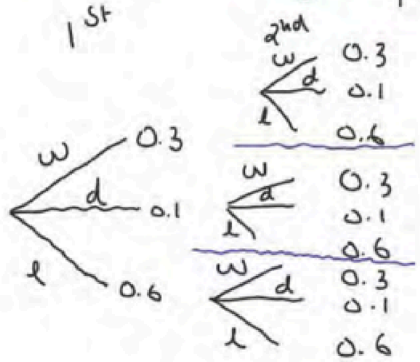
Work out the probability that Sally will win **exactly** one of the two games against Martin.

MODEL ANSWER

$P(w) + P(d) + P(l) = 1$

$0.3 + 0.1 + p(l) = 1 \rightarrow p(l) + 0.4 = 1 \quad (-0.4)$

$p(l) = 0.6$



$P(\text{exactly } 1) = 0.3 \times 0.1 + 0.3 \times 0.6 + 0.1 \times 0.3 + 0.6 \times 0.3$

$= 0.03 + 0.18 + 0.03 + 0.18$

$= 0.42$

0.03
0.18
0.03
0.18
0.42
2

0.42

QUESTION

The straight line L_1 has equation $y = 3x - 4$
The straight line L_2 is perpendicular to L_1 and passes through the point (9, 5)

Find an equation of line L_2

MODEL ANSWER

$m_{L_1} \times m_{L_2} = -1$

$3 \times m_{L_2} = -1$

$m_{L_2} = -\frac{1}{3}$ ✓1

$y = -\frac{1}{3}x + c$

at $x = 9$, $y = 5$

$5 = -\frac{1}{3} \times 9 + c$ ✓2

$5 = -3 + c$

$5 = -3 + c$) $+3$

$+3$ ($c = 8$

$y = -\frac{1}{3}x + 8$ ✓3

QUESTION

Shirley wants to find an estimate for the number of bees in her hive.

On Monday she catches 90 of the bees.
She puts a mark on each bee and returns them to her hive.

On Tuesday she catches 120 of the bees.
She finds that 20 of these bees have been marked.

(a) Work out an estimate for the total number of bees in her hive.

Shirley assumes that none of the marks had rubbed off between Monday and Tuesday.

(b) If Shirley's assumption is wrong, explain what effect this would have on your answer to part (a).

MODEL ANSWER

M : T   M : T
20 : 120   90 : n

$$\frac{20}{120} = \frac{90}{n} \quad \checkmark_{1,2}$$

$\times n$ (both sides):

$$\frac{20n}{120} = 90$$

$$20n = 90 \times 120$$

$\div 20$:

$$n = \frac{90 \times 120}{20}$$

$$n = 90 \times 6 = 540 \quad \checkmark_3$$

540

fewer marked bees. This means the answer will be over-estimated. ✓

QUESTION

Make f the subject of the formula $d = \dfrac{3(1 - f)}{f - 4}$

MODEL ANSWER

7 Make f the subject of the formula $d = \dfrac{3(1-f)}{f-4}$

$\times(f-4)$

$$(f-4)d = 3(1-f)$$

$$fd - 4d = 3 - 3f$$

$$fd + 3f = 4d + 3$$

$$f(d+3) = 4d + 3$$

$\div(d+3)$

$$f = \frac{4d+3}{d+3}$$

QUESTION

x is proportional to $\sqrt{y}$ where $y > 0$

y is increased by 44%

Work out the percentage increase in x.

MODEL ANSWER

x is proportional to $\sqrt{y}$ where $y > 0$ $\rightarrow x = K\sqrt{y}$ ✓1

y is increased by 44% $y_n = y \times 1.44$

Work out the percentage increase in x.

$\hookrightarrow x_n = K \times \sqrt{y \times 1.44}$ ✓2 $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

$x_n = K \times \sqrt{y} \times \sqrt{1.44}$

$x_n = K\sqrt{y} \times 1.2$

$x_n = x \times 1.2 \rightarrow$ 20% increase ✓3

QUESTION

f and g are functions such that

$$f(x) = \frac{12}{\sqrt{x}} \quad \text{and} \quad g(x) = 3(2x + 1)$$

(a) Find g(5)

(b) Find gf(9)

(c) Find $g^{-1}(6)$

MODEL ANSWER

↳ Substitute 5 for x in g

$g(5) = 3(2\times5+1)$

$= 3(11) = 33$

(b) Find gf(9)

$f(x) = \frac{12}{\sqrt{x}}$ $g(x) = 3(2x+1)$

$g(f(9))$

$f(9) = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$ ✓1

$g(f(9)) = g(4) = 3(2\times4+1)$

$= 27$

(c) Find $g^{-1}(6)$ $g(x) = 3(2x+1)$

① finding $g^{-1}(x)$

$\rightarrow x = 3(2y+1)$ (rearrange for y)

$\div 3$: $\frac{x}{3} = 2y+1$

-1: $\frac{x}{3} - 1 = 2y$

$\div 2$: $y = \frac{1}{2}\left(\frac{x}{3}-1\right)$

$\therefore g^{-1}(x) = \frac{1}{2}\left(\frac{x}{3}-1\right)$ ✓1

$\rightarrow g^{-1}(6) = \frac{1}{2}\left(\frac{6}{3}-1\right)$

$= \frac{1}{2}\times1 = \frac{1}{2}$

½ ✓1

(2)

(Total for Question 10 is 5 marks)

QUESTION

Show that $\dfrac{\sqrt{180} - 2\sqrt{5}}{5\sqrt{5} - 5}$ can be written in the form $a + \dfrac{\sqrt{5}}{b}$ where a and b are integers.

MODEL ANSWER

$$\sqrt{180} = \sqrt{9\times 20}$$
$$= \sqrt{9}\times\sqrt{20}$$
$$= 3\times\sqrt{20}$$
$$= 3\times\sqrt{4\times 5}$$
$$= 3\times\sqrt{4}\times\sqrt{5}$$
$$= 3\times 2\times\sqrt{5} = 6\sqrt{5} \quad ✓_1$$

$$\frac{a}{b-c} = \frac{a(b+c)}{(b-c)(b+c)}$$
$$b^2 - c^2$$

$$\sqrt{ab} = \sqrt{a}\times\sqrt{b}$$

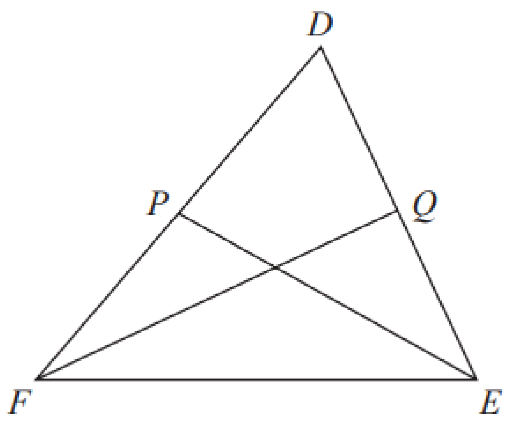
$$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$$

$$\frac{6\sqrt{5} - 2\sqrt{5}}{5\sqrt{5} - 5} = \frac{4\sqrt{5}}{5\sqrt{5} - 5}$$
$$= \frac{4\sqrt{5}(5\sqrt{5} + 5)}{(5\sqrt{5} - 5)(5\sqrt{5} + 5)} \quad ✓_2$$
$$= \frac{100 + 20\sqrt{5}}{125 - 25}$$
$$= \frac{100 + 20\sqrt{5}}{100} \quad ✓_3$$
$$= \frac{100}{100} + \frac{20\sqrt{5}}{100}$$
$$= 1 + \frac{\sqrt{5}}{5} \quad ✓_4$$

$$\therefore a = 1$$
$$b = 5$$

QUESTION

DEF is a triangle.



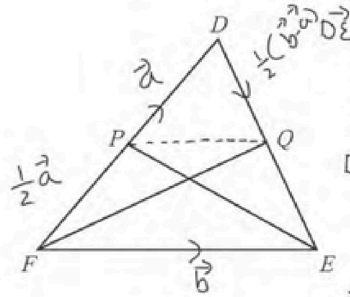
P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

MODEL ANSWER

DEF is a triangle.



P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

$\overrightarrow{DE} = \overrightarrow{DF} + \overrightarrow{FE}$

$= -\vec{a} + \vec{b}$

$= \vec{b} - \vec{a}$

$\overrightarrow{DQ} = \frac{1}{2}\overrightarrow{DE}$

$= \frac{1}{2}(\vec{b} - \vec{a})$ ✓1

$\overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DQ}$

$= \frac{1}{2}\vec{a} + \frac{1}{2}(\vec{b} - \vec{a})$ ✓2

$= \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$

$= \frac{1}{2}\vec{b}$ ✓3

for 2 vectors to be parallel, they must be scalar multiples of eachother.

$\overrightarrow{PQ} = \frac{1}{2}\vec{b}$ and $\overrightarrow{FE} = \vec{b}$

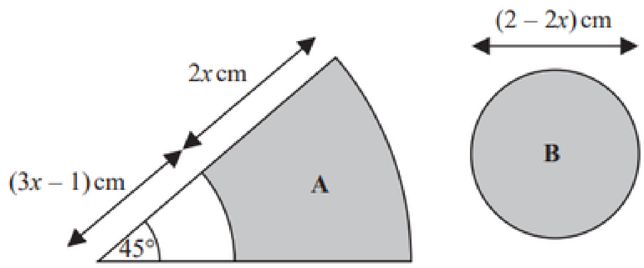
$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{FE}$ and so $\overrightarrow{PQ}$ is a scalar multiple of $\overrightarrow{FE}$.

Therefore, $\overrightarrow{PQ}$ and $\overrightarrow{FE}$ are parallel, as required. ✓4

QUESTION

The diagram shows two shaded shapes, **A** and **B**.

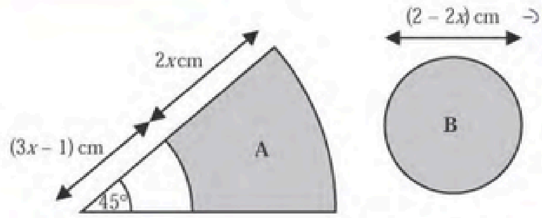
Shape **A** is formed by removing a sector of a circle with radius $(3x - 1)$ cm from a sector of the circle with radius $(5x - 1)$ cm.
Shape **B** is a circle of diameter $(2 - 2x)$ cm.



The area of shape **A** is equal to the area of shape **B**.

Find the value of x.
You must show all your working.

MODEL ANSWER



$\Rightarrow r = \frac{1}{2}(2-2x)$

$r = 1-x$

The area of shape A is equal to the area of shape B.

$a.s = \frac{\theta}{360} \times \pi r^2$

Find the value of x.

You must show all your working.

Area of shape A = area of sector − cutout.

$\text{area of sector} = \frac{45°}{360°} \times \pi \times (5x-1)^2 \text{ cm}^2$

$= \frac{1}{8} \times \pi \times (5x-1)^2$

$\text{cutout} = \frac{45°}{360°} \times \pi \times (3x-1)^2 \text{ cm}^2$

$= \frac{1}{8} \times \pi (3x-1)^2$

$(5x-1)(5x-1)$

$25x^2 - 5x - 5x + 1$

$= \frac{1}{8}\pi\left((5x-1)^2 - (3x-1)^2\right)$ ✓1

$(3x-1)(3x-1)$

$9x^2 - 3x - 3x + 1$

$= \frac{1}{8}\pi\left((25x^2 - 10x + 1) - (9x^2 - 6x + 1)\right)$

$= \frac{1}{8}\pi\left(16x^2 - 4x\right)$ ✓2

$(1-x)(1-x)$

$1 - x - x + x^2$

$\text{area of B} = \pi(1-x)^2 = \pi(x^2 - 2x + 1)$

$x_1, x_2 = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2 \times 2}$

$= \frac{1}{2}, -2$ ✓4

$\frac{1}{8}(16x^2 - 4x)\pi = \pi(x^2 - 2x + 1)$) ×8

$16x^2 - 4x = 8x^2 - 16x + 8$) $-(8x^2 - 16x + 8)$

$8x^2 + 12x - 8 = 0$

$2x^2 + 3x - 2 = 0$ ✓3

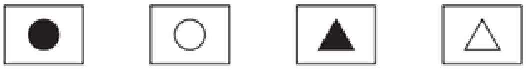
$x = \frac{1}{2}$ ✓5

(Total for Question 22 is 5 marks)

QUESTION

There are four types of cards in a game.

Each card has a black circle or a white circle or a black triangle or a white triangle.



number of cards with a black shape	:	number of cards with a white shape	=	3 : 5
number of cards with a circle	:	number of cards with a triangle	=	2 : 7

Express the total number of cards with a black shape as a fraction of the total number of cards with a triangle.

MODEL ANSWER

Express the total number of cards with a black shape as a fraction of the total number of cards with a triangle.

↳ $\frac{\text{b.shape}}{\text{triangle}} = \frac{3/8}{7/9}$ ✓2 $= \frac{3}{8} \times \frac{9}{7} = \frac{27}{56}$

1/ fraction for b.shapes $= \frac{3}{8}$ ✓1   2/ fraction for triangle $= 7/9$

JUNE 2022

QP and MA

HIGHER

QUESTION

Express $0.1\dot{1}\dot{7}$ as a fraction.
You must show all your working.

MODEL ANSWER

Educate Cloud

let $x = 0.1\dot{1}\dot{7}$ ✓ (1)

$x = 0.1171717\ldots$

get rid of the repeating decimals

$100x = 11.71717$ – subtract

$x = 0.11717$

✓ (1)

$99x = 11.6$

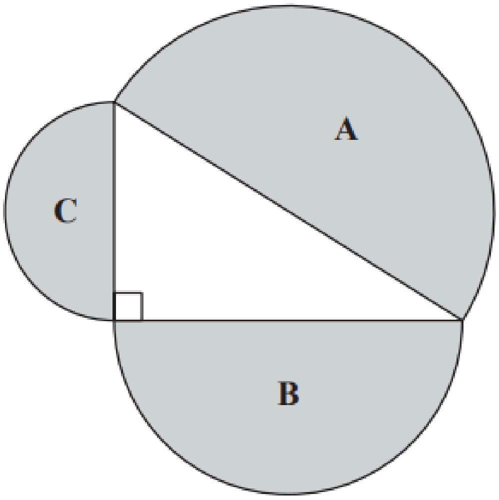
✓ (1)

$x = \frac{11.6}{99} = \frac{116}{990}$

$\frac{116}{990}$

QUESTION

A right-angled triangle is formed by the diameters of three semicircular regions, **A**, **B** and **C** as shown in the diagram.



Show that

area of region **A** = area of region **B** + area of region **C**

MODEL ANSWER

Educate Cloud

right angled △ : lengths $a^2 + b^2 = c^2$ ✓ (1)

area B : $\frac{1}{2} \times \pi \times \left(\frac{a}{2}\right)^2 = \frac{a^2\pi}{8}$

area C : $\frac{1}{2} \times \pi \times \left(\frac{b}{2}\right)^2 = \frac{b^2\pi}{8}$

area B+C : $\frac{a^2\pi}{8} + \frac{b^2\pi}{8}$ ✓ (1)

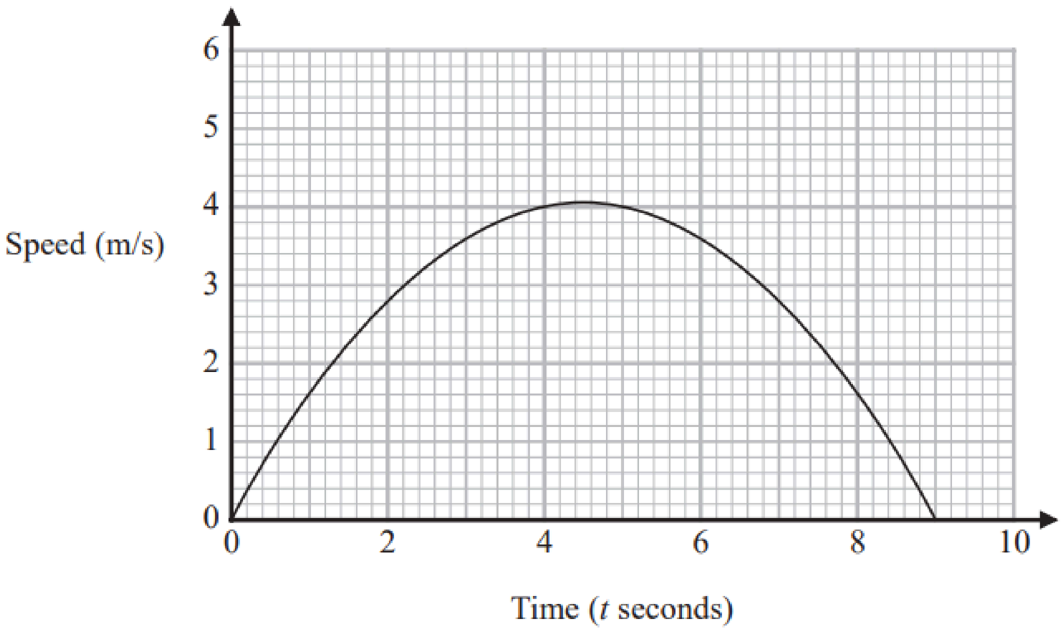
factor out $\frac{\pi}{8}(a^2 + b^2)$

$a^2+b^2 = c^2$, so substitute $\frac{\pi}{8} c^2$

area A : $\frac{1}{2} \times \pi \times \left(\frac{c}{2}\right)^2 = \frac{\pi}{8} c^2$ ✓ (1) → they match

Question

Here is a speed-time graph.



(a) Work out an estimate of the gradient of the graph at $t = 2$

(b) What does the area under the graph represent?

MODEL ANSWER

Educate Cloud

(a) Work out an estimate of the gradient of the graph at $t = 2$

tangent at $t=2$

$$\frac{\text{change in } y}{\text{change in } x} = \frac{4.8 - 1}{4 - 0} = 0.95$$ ✓ (1)

✓ (1)

0.95

(3)

(b) What does the area under the graph represent?

speed × time = distance travelled

✓ (1)

(1)

Question

A, B and C are three points such that

$$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b}$$

$$\overrightarrow{AC} = 15\mathbf{a} + 20\mathbf{b}$$

(a) Prove that A, B and C lie on a straight line.

D, E and F are three points on a straight line such that

$$\overrightarrow{DE} = 3\mathbf{e} + 6\mathbf{f}$$

$$\overrightarrow{EF} = -10.5\mathbf{e} - 21\mathbf{f}$$

(b) Find the ratio

length of DF : length of DE

MODEL ANSWER

(a) Prove that A, B and C lie on a straight line.

factor and common point

$\overrightarrow{AB} = 3a + 4b$

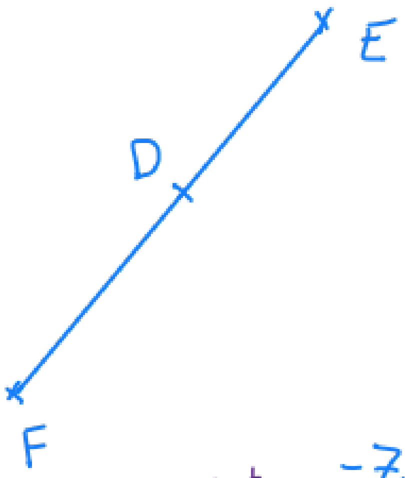
$\overrightarrow{AC} = 5(3a + 4b)$ ✓①

• $\overrightarrow{AC}$ is a multiple of $\overrightarrow{AB}$ and they share point A ✓①

(2)

(b) Find the ratio

length of DF : length of DE



$\overrightarrow{DF} = \overrightarrow{DE} + \overrightarrow{EF}$

$= 3e + 6f + -10.5e + -21f$

collect e and f

$= -7.5e - 15f$ ✓①

$\overrightarrow{DF} : \overrightarrow{DE}$

$-7.5e - 15f : 3e + 6f$

factor out ↓

$-2.5(3e + 6f) : 3e + 6f$

$\div 3e + 6f$ $\quad\quad$ $\div 3e+6f$

$-2.5 : 1$ ✓①

negative answer just means opposite direction → $-5 : 2$

5 : 2 ✓①

(3)

(Total for Question 15 is 5 marks)

Question

A first aid test has two parts, a theory test and a practical test.
The probability of passing the theory test is 0.75
The probability of passing only one of the two parts is 0.36

The two events are independent.

Work out the probability of passing the practical test.

MODEL ANSWER

Educate Cloud

P pass theory $= 0.75$

fail theory $= 1 - 0.75 = 0.25$ ✓ (1)

Pass one of 2 parts $= 0.36$

pass theory, fail practical

$0.75 \times y$ let fail practical $= y$

pass practical, fail theory

$(1-y) \times 0.25$ ✓ (1)

$0.75y + 0.25(1-y) = 0.36$

$0.75y + 0.25 - 0.25y = 0.36$ ✓ (1)

collect terms

$0.5y = 0.11$

$y = \frac{0.11}{0.5} = 0.22$

y was to fail practical

pass $= 1 - 0.22 = 0.78$

0.78 ✓ (1)

(Total for Question 16 is 4 marks)

Question

y is directly proportional to the square root of t.
$y = 15$ when $t = 9$

t is inversely proportional to the cube of x.
$t = 8$ when $x = 2$

Find a formula for y in terms of x.
Give your answer in its simplest form.

MODEL ANSWER

Educate Cloud

① $y = k\sqrt{t}$

$15 = k\sqrt{9}$

$15 = 3k$ so $k = 5$ => $y = 5\sqrt{t}$ ✓ ①

② $t = \frac{K}{x^3}$

$8 = \frac{K}{2^3}$ so $K = 64$ => $t = \frac{64}{x^3}$ ✓ ①

y in terms of x, substitute out t

$y = 5\sqrt{t}$

$y^2 = 25t$

$t = \frac{y^2}{25}$

$\frac{y^2}{25} = \frac{64}{x^3}$ ✓ ①

$y^2 = \frac{64 \times 25}{x^3}$

$y = \frac{\sqrt{64 \times 25}}{\sqrt{x^3}}$

$y = \frac{8 \times 5}{\sqrt{x^3}} = \frac{40}{\sqrt{x^3}}$ ✓ ①

(Total for Question 17 is 4 marks)

Question

3 Work out the value of $\dfrac{\left(5\frac{4}{9}\right)^{-\frac{1}{2}} \times \left(4\frac{2}{3}\right)}{2^{-3}}$

You must show all your working.

MODEL ANSWER

Educate Cloud

simplify fractions first

$$\frac{\left(\frac{49}{9}\right)^{-1/2} \times \frac{14}{3}}{2^{-3}}$$

a − power flips the fraction

✓ ①

$$\left(\frac{3}{7} \times \frac{14}{3}\right) \div \frac{1}{8}$$ ✓ ①

$$\frac{\cancel{3} \times 14}{7 \times \cancel{3}} \times 8$$ ✓ ①

3's cancel
7's cancel

left with 2×8
$= 16$

16 ✓ ①

Question

Solve $\frac{1}{2x-1} + \frac{3}{x-1} = 1$

Give your answer in the form $\frac{p \pm \sqrt{q}}{2}$ where p and q are integers.

MODEL ANSWER

Educate Cloud

$$\frac{1}{2x-1} + \frac{3}{x-1} = 1$$

cross multiply ✓①

$$\frac{1(x-1) + 3(2x-1)}{(2x-1)(x-1)} = 1$$

$$x-1 + 6x - 3 = (2x-1)(x-1)$$

$$7x - 4 = 2x^2 - 3x + 1$$

$$0 = 2x^2 - 10x + 5 \quad ✓①$$

quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\frac{10 \pm \sqrt{(-10)^2 - (4 \times 2 \times 5)}}{2 \times 2} \quad ✓①$$

$$= \frac{10 \pm \sqrt{100 - 40}}{4}$$

$$\frac{10 \pm \sqrt{60}}{4}$$

÷2

roots can only ÷ with roots. Have to do $\sqrt{60} \div \sqrt{4} = \sqrt{15}$

$$\frac{5 \pm \sqrt{15}}{2} \quad ✓①$$

$$\frac{5 \pm \sqrt{15}}{2}$$

(Total for Question 19 is 4 marks)

Question

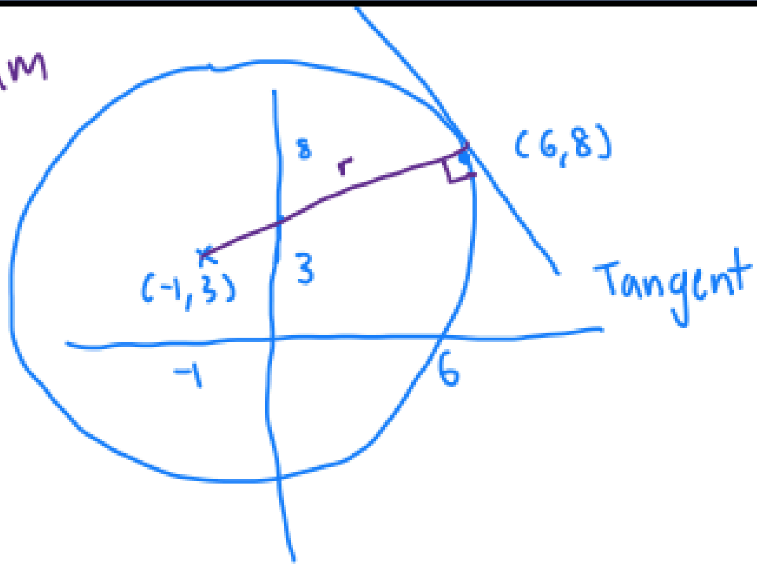
The centre of a circle is the point with coordinates $(-1, 3)$

The point A with coordinates $(6, 8)$ lies on the circle.

Find an equation of the tangent to the circle at A.
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

MODEL ANSWER

Diagram



the gradients of the radius and tangent are perpendicular

$m_{radius} \rightarrow \dfrac{\Delta y}{\Delta x} = \dfrac{8-3}{6--1} = \dfrac{5}{7}$ ✓①

$m_{TANGENT} \rightarrow -\dfrac{7}{5}$ ✓① (negative reciprocal of radius)

$y - y_1 = m(x - x_1)$ → tangent hits $(6, 8)$ (x_1, y_1)

$y - 8 = -\dfrac{7}{5}(x-6)$ ✓①

$5y - 40 = -7(x-6)$

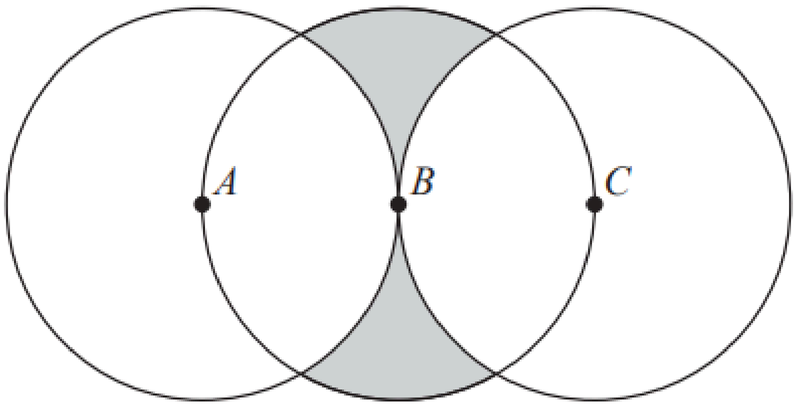
$5y - 40 = -7x + 42$

$7x + 5y - 82 = 0$ ✓①

Question

The diagram shows three circles, each of radius 4 cm.

The centres of the circles are A, B and C such that ABC is a straight line and $AB = BC = 4$ cm.



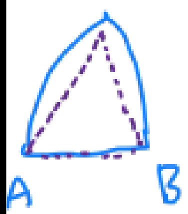
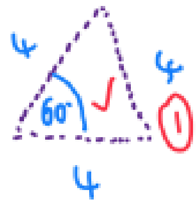
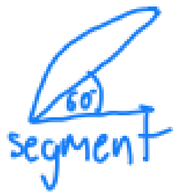
Work out the total area of the two shaded regions.
Give your answer in terms of π

MODEL ANSWER

Work out the total area of the two shaded regions.
Give your answer in terms of π

shaded = middle circle − white areas

white areas ⇒ 4 × 

 split into  ✓ (1) and 2 × 

triangle area $= \frac{1}{2} \times 4 \times 4 \times \sin 60 \left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$ ✓ (1)

one segment $= 4^2\pi \times \frac{60°}{360°} - \frac{1}{2} \times 4 \times 4 \times \sin 60$ ✓ (1)

one white area ⇒ $4\sqrt{3} + 2\left(\frac{16\pi}{6} - 4\sqrt{3}\right)$ (one Δ + 2 segments)

white area $= \frac{16\pi}{3} - 4\sqrt{3}$ ✓ (1)

Shaded ⇒ middle circle − 4 × white area

$16\pi - 4\left(\frac{16\pi}{3} - 4\sqrt{3}\right) = 16\sqrt{3} - \frac{16\pi}{3}$ cm² ✓ (1)

(Total for Question 21 is 5 marks)

JUNE 2023

QP and MA

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Question

The equation of line $\mathbf{L}_1$ is $y = 2x - 5$
The equation of line $\mathbf{L}_2$ is $6y + kx - 12 = 0$

$\mathbf{L}_1$ is perpendicular to $\mathbf{L}_2$

Find the value of k.
You must show all your working.

MODEL ANSWER

rewrite both lines in $y = mx + c$ form

$L_1 \Rightarrow \quad y = 2x - 5$

$L_2 \Rightarrow \quad 6y + kx - 12 = 0$

$-kx \quad +12 \qquad -kx + 12$

$6y = -kx + 12$

$\div 6 \qquad \div 6$

$y = -\frac{k}{6} + 2$ ✓(1)

Perpendicular Lines : gradients are negative reciprocals

L_1 gradient $= 2$

L_2 gradient has to be $-\frac{1}{2}$ ✓(1)

$-\frac{1}{2} = -\frac{k}{6}$ ($\times 6$, $\times 6$)

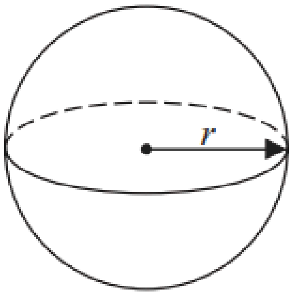
$-3 = -k$

$3 = k$

$k = $ 3 ✓(1)

Question

Here is a sphere.



Surface area of sphere = $4\pi r^2$

$\frac{3}{8}$ of the surface area of this sphere is $75\pi\,\text{cm}^2$

Find the diameter of the sphere.
Give your answer in the form $a\sqrt{b}$ where a is an integer and b is a prime number.

MODEL ANSWER

Educate Cloud

$\frac{3}{8}$ of surface area $= 75\pi$

$\div 3$ $\qquad \div 3$

$\frac{1}{8}$ of surface area $= 25\pi$

$\times 8$ $\qquad \times 8$

Whole surface area $= 200\pi$ ✓(1)

$4\pi r^2 = 200\pi$

cancel π's and divide by 4 ✓(1)

$r^2 = 50$ ✓(1) so $r = \sqrt{50}$

diameter $= 2r = 2\sqrt{50}$ or $2\sqrt{25\times 2}$

$= 10\sqrt{2}$

✓(1)

$10\sqrt{2}$ cm

(Total for Question 16 is 4 marks)

Questions

Make x the subject of the formula $y = \dfrac{4(2x-7)}{5x+3}$

MODEL ANSWER

multiply both sides by the denominator

$y(5x+3) = 4(2x-7)$ ✓(1)

expand : $5xy + 3y = 8x - 28$

$-8x \quad -8x$

$5xy - 8x + 3y = -28$

$-3y \quad -3y$

$5xy - 8x = -28 - 3y$ ✓(1)

get x's on one side

factor out the x

$x(5y - 8) = -28 - 3y$ ✓(1)

divide by $5y-8$

$x = \frac{-28-3y}{5y-8}$

✓(1)

$x = \frac{-28-3y}{5y-8}$

(Total for Question 17 is 4 marks)

Questions

7 kg of carrots and 5 kg of tomatoes cost a total of 480p

cost of 1 kg of carrots : cost of 1 kg of tomatoes = 5 : 9

Work out the cost of 1 kg of carrots and the cost of 1 kg of tomatoes.

MODEL ANSWER

Educate Cloud

set an equation ① $7c + 5t = 480$ with weights ✓①

set another equation with ratios $\begin{matrix} c : t \\ 5 : 9 \end{matrix}$ so ② $9c = 5t$ ✓①

Substitute ① into ②

$7c + 9c = 480$

$16c = 480$

$c = 30$

① $7c + 5t = 480$

$c = 30$

$210 + 5t = 480$

$5t = 270$

$t = 54$ ✓

✓①

carrots ....30........p

tomatoes ....54........p ✓①

Questions

The menu in a restaurant has starters, main courses and desserts.

There are 5 starters.
There are 12 main courses.
There are x desserts.

There are 420 different ways to choose one starter, one main course and one dessert.

Work out the value of x.

MODEL ANSWER

Educate Cloud

$5 \times 12 \times x = 420$

$60x = 420$ ✓ (1)

$x = 7$

✓ (1)

$x =$ 7

Questions

For $x \geqslant 0$, the functions f and g are such that

$$f(x) = 3x + 4 \qquad g(x) = \frac{\sqrt{x} + 2}{5}$$

(a) Find $g^{-1}(x)$

$g^{-1}(x) =$ ....

(2)

(b) Solve $gf(x) = 3$

MODEL ANSWER

Educate Cloud

(a) Find $g^{-1}(x)$

let $y = g(x)$ $\quad y = \frac{\sqrt{x} + 2}{5}$ ✓ (1)

$\times 5 \quad \times 5$

solve for x

$5y = \sqrt{x} + 2$

$-2 \quad -2$

$5y - 2 = \sqrt{x}$

$(5y-2)^2 = x$

change y to x and x to $g^{-1}(x)$

$(5x-2)^2 = g^{-1}(x)$

$g^{-1}(x) = (5x-2)^2$ ✓ (1)

(2)

(b) Solve $gf(x) = 3$

use $g(x) = \frac{\sqrt{x} + 2}{5}$ but substitute $x = 3x + 4$

$\frac{\sqrt{3x+4} + 2}{5} = 3$ ✓ (1)

$\times 5 \quad \times 5$

$\sqrt{3x+4} + 2 = 15$

$-2 \quad -2$

$\sqrt{3x+4} = 13$

$3x + 4 = 169$ ✓ (1)

$-4 \quad -4$

$3x = 165$ so $x = 55$

$\div 3 \quad \div 3$

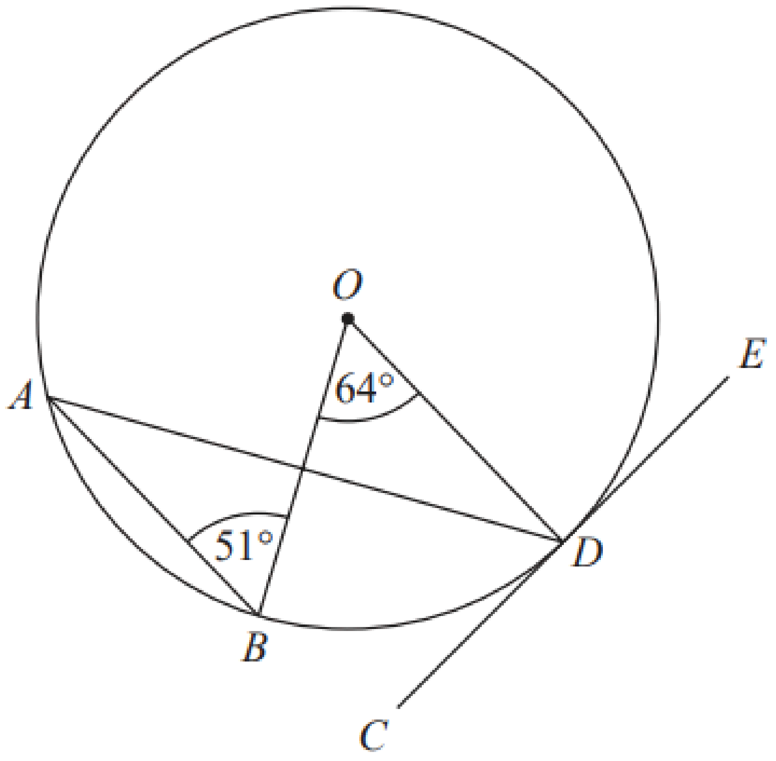
$x = 55$ ✓ (1)

(3)

(Total for Question 20 is 5 marks)

Questions

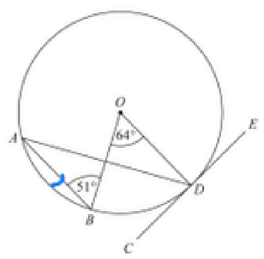
A, B and D are points on a circle with centre O.
CDE is the tangent to the circle at D.



Work out the size of angle ADC.
Write down any circle theorems you use.

MODEL ANSWER

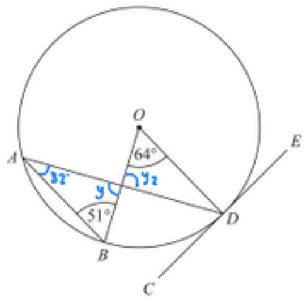
Educate Cloud



angle BAD = $\frac{1}{2}$ angle BOD

because angle at circumference is half the angle at centre ✓ (1)

angle BAD = 32° ✓ (1)



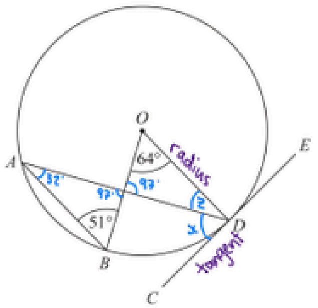
let y = angle inside the triangle

$y = 180 - 32 - 51$
$= 97°$

because angles in a Δ sum to 180°

$y_2 = 97°$

because vertically opposite angles are equal



angle z = 180 − 97 − 64 ✓ (1)
= 19°

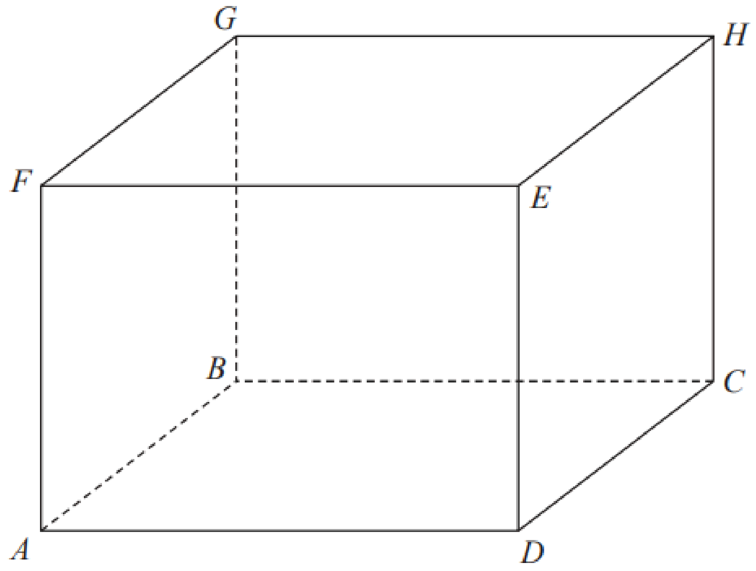
because angles in a Δ sum to 180°

angle x = 90° − 19
= 71°

because a radius (OD) and tangent meeting form 90° ✓ ...... 71 ✓ (1)

Question

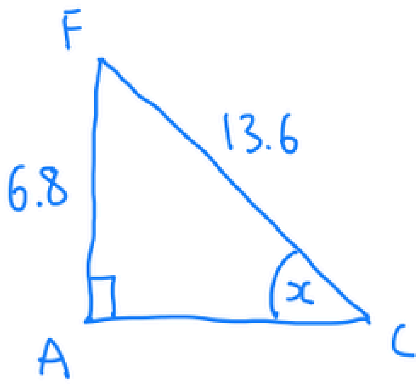
ABCDEFGH is a cuboid.



$AF = 6.8$ cm
$FC = 13.6$ cm

Work out the size of the angle between *FC* and the plane *ABCD*.

MODEL ANSWER



SOH

$$\sin x = \frac{6.8}{13.6}$$ (simplifies to $\frac{1}{2}$)

$$x = \sin^{-1}\left(\frac{1}{2}\right)$$ ✓ (1)

$$x = 30°$$

Question

Write $\frac{3\sqrt{3}}{4-\sqrt{3}} - \frac{2}{\sqrt{3}}$ in the form $\frac{a\sqrt{3}+b}{c}$ where a, b and c are integers.

MODEL ANSWER

Educate Cloud

cross multiply to get the same denominator

$$\frac{3\sqrt{3}(\sqrt{3}) - 2(4-\sqrt{3})}{(4-\sqrt{3})(\sqrt{3})} \text{ expands to } \frac{9-8+2\sqrt{3}}{4\sqrt{3}-3} \quad ✓①$$

$$\frac{1+2\sqrt{3}}{4\sqrt{3}-3} \times \frac{4\sqrt{3}+3}{4\sqrt{3}+3} \text{ (rationalise)} \quad \frac{(1+2\sqrt{3})(4\sqrt{3}+3)}{(4\sqrt{3}-3)(4\sqrt{3}+3)} \quad ✓① \quad \text{expands to } \frac{4\sqrt{3}+3+24+6\sqrt{3}}{48-9} \quad ✓①$$

collect like terms : $\frac{10\sqrt{3}+27}{39}$ in $\frac{a\sqrt{3}+b}{c}$ form

Question

Find the set of possible values of x for which

$$4x^2 - 25 < 0 \quad \textbf{and} \quad 12 - 5x - 3x^2 > 0$$

You must show all your working.

MODEL ANSWER

$4x^2 - 25 < 0$

$+25 \quad +25$

$4x^2 < 25$

$\div 4 \quad \div 4$

$x^2 < \frac{25}{4}$ ✓(1)

square root
could be positive or negative (sign change)

$x < \frac{5}{2} \qquad x > -\frac{5}{2}$

↓

$-\frac{5}{2} < x < \frac{5}{2}$

or

$-2.5 < x < 2.5$ ✓(1)

$12 - 5x - 3x^2 > 0$

multiply by -1 and flip sig

$3x^2 + 5x - 12 < 0$

numbers that × to get -36 are +9 and -4
+ to get +5

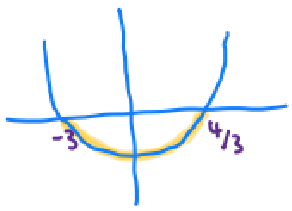
rewrite as $3x^2 + 9x - 4x - 12 < 0$

$3x(x+3) - 4(x+3) < 0$

match

$(x+3)(3x-4) < 0$ ✓(1)

graph



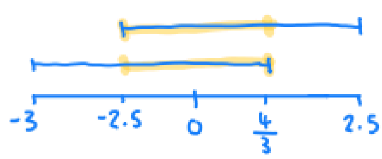
0 is greater so under the x axis

$-3 < x < \frac{4}{3}$ ✓(1)

To fit both inequalities

$-2.5 < x < 2.5$

$-3 < x < \frac{4}{3}$



shaded : $-2.5 < x < \frac{4}{3}$ ✓

✓(1)

$-2.5 < x < \frac{4}{3}$

JUNE 2024

QP and MA

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Question

Expand and simplify $(3x - 1)(2x + 3)(x - 5)$

MODEL ANSWER

Educate Cloud

Expansion of first 2 terms :

$$(3x-1)(2x+3) = 6x^2 + 9x - 2x - 3 \quad (1)$$
$$= 6x^2 + 7x - 3$$

Expansion with the final term :

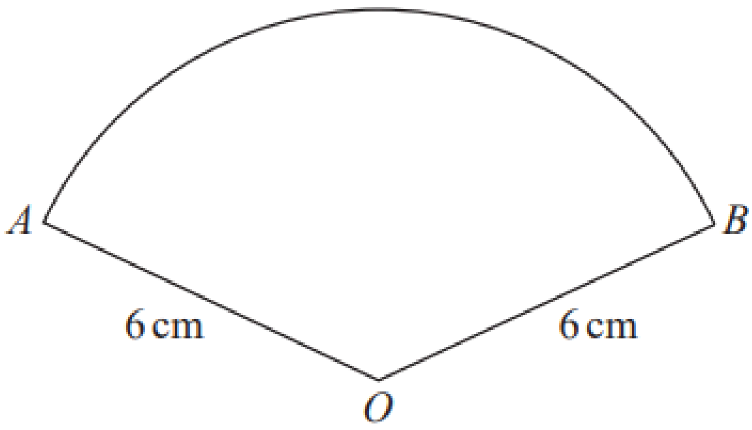
$$(6x^2 + 7x - 3)(x-5)$$

$$= 6x^3 - 30x^2 + 7x^2 - 35x - 3x + 15 \quad (1)$$

$$= 6x^3 - 23x^2 - 38x + 15 \quad (1)$$

Question

OAB is a sector of a circle with centre O and radius 6 cm.



The length of the arc AB is 5π cm.

Work out, in terms of π, the area of the sector.
Give your answer in its simplest form.

MODEL ANSWER

Educate Cloud

$$\text{arc } AB = \frac{\theta}{360^\circ} \times 2\pi r \quad , \text{ where } r = 6$$

$$5\pi = \frac{\theta}{360^\circ} \times 12\pi \quad (1)$$

$$\theta = \frac{5\pi}{12\pi} \times 360^\circ = 150^\circ \quad (1)$$

$$\text{Area of sector} = \frac{150}{360^\circ} \times \pi \times 6^2 \quad (1)$$

$$= \frac{5}{12_1} \times \pi \times 36^3$$

$$= 15\pi \quad (1)$$

Question

There are only n orange sweets and 1 white sweet in a bag.

Saira takes at random a sweet from the bag and eats the sweet.
She then takes at random another sweet from the bag and eats this sweet.

Show that the probability that Saira eats two orange sweets is $\frac{n-1}{n+1}$

MODEL ANSWER

Educate Cloud

Total number of sweets : $n+1$

1st take : P(orange sweet) $= \frac{n}{n+1}$

2nd take : P (orange sweet) $= \frac{n-1}{(n+1)-1} = \frac{n-1}{n}$ (1)

include -1 because she already eats 1 orange sweet during first event

P(orange sweet on both takes) $= \left(\frac{n}{n+1}\right) \times \left(\frac{n-1}{n}\right)$ (1)

$= \frac{n-1}{n+1}$

(Total for Question 16 is 2 marks)

Question

(a) Rationalise the denominator of $\frac{1}{\sqrt{7}}$

(b) Simplify fully $\sqrt{80} - \sqrt{5}$

MODEL ANSWER

Educate Cloud

(a) Rationalise the denominator of $\frac{1}{\sqrt{7}}$

$$\frac{1 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7}$$

since $\sqrt{7} \times \sqrt{7} = 7$,
we rationalise the denominator to equal to 7.

$\frac{\sqrt{7}}{7}$ (1)

(1)

(b) Simplify fully $\sqrt{80} - \sqrt{5}$

$\sqrt{80} - \sqrt{5}$

$= \sqrt{16 \times 5} - \sqrt{5}$

$= (\sqrt{16} \times \sqrt{5}) - \sqrt{5}$ (1)

$= 4\sqrt{5} - \sqrt{5}$

$= 3\sqrt{5}$ (1)

$3\sqrt{5}$

(2)

Question

Show that $0.\dot{1}\dot{5} + 0.2\dot{2}\dot{7}$ can be written in the form $\dfrac{m}{66}$ where m is an integer.

MODEL ANSWER

Educate Cloud

$0.\dot{1}\dot{5} = 0.1515....$

$0.2\dot{2}\dot{7} = 0.22727.....$

$0.\dot{1}\dot{5} + 0.2\dot{2}\dot{7} = 0.37878.....$ (1)

if $x = 0.37878.....$

$10x = 0.37878 \times 10$

$= 3.7878....$

$1000x = 0.37878 \times 1000$

$= 378.78.....$

$1000x - 10x = 378.78... - 3.7878....$

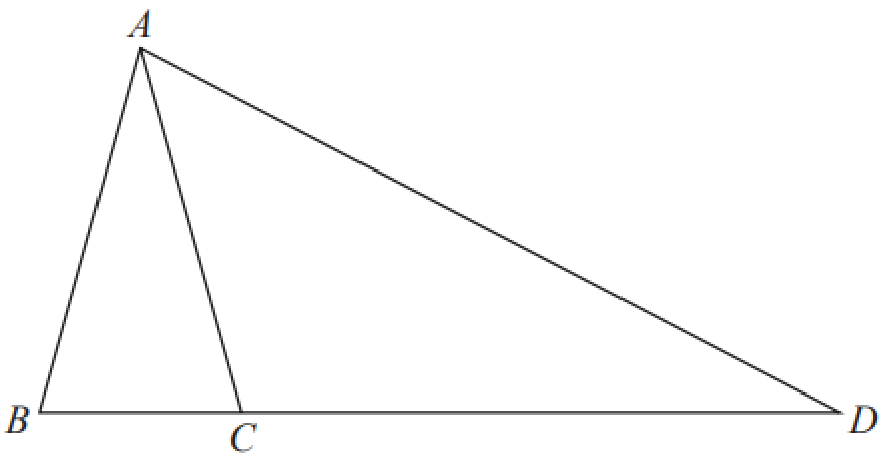
$990x = 375$

$x = \frac{375}{990}$ (1)

Simplification of x :

$\frac{375 \div 5}{990 \div 5} = \frac{75 \div 3}{198 \div 3} = \frac{25}{66}$ (1)

Question



ABC and DAB are similar isosceles triangles.

$AB = AC$
$AD = BD$

$BC : CD = 4 : 21$

Find the ratio $AB : AD$

MODEL ANSWER

Educate Cloud

Since both triangles are similar :

$$\frac{AB}{BC} = \frac{AD}{AB}$$

$$\frac{AB}{4} = \frac{25}{AB} \quad (1)$$

slanted height (AB), base (4); slanted height (25), base (AB)

$$AB^2 = 25 \times 4$$

$$\sqrt{AB^2} = \sqrt{100}$$

$$AB = 10$$

$$\therefore \frac{AD}{AB} = \frac{25 \div 5}{10 \div 5}$$

common factor

$$= \frac{5}{2} \quad (1)$$

2 : 5

$\therefore AB : AD = 2 : 5$ (1)

(Total for Question 10 is 3 marks)

Question

$$2^x = \frac{2^n}{\sqrt[3]{2}} \qquad 2^y = \left(\sqrt{2}\right)^5$$

Given that $x + y = 8$

work out the value of n.

MODEL ANSWER

$2^x = \frac{2^n}{2^{1/3}} = 2^{n-\frac{1}{3}}$ (1)

$2^y = 2^{(\frac{1}{2})\times 5} = 2^{\frac{5}{2}}$

$x = n - \frac{1}{3}$

(1)

$y = \frac{5}{2}$

$\therefore\ x + y = 8$

$\left(n - \frac{1}{3}\right) + \frac{5}{2} = 8$

$n - \frac{1}{3} = 8 - \frac{5}{2}$

$n = 8 - \frac{5}{2} + \frac{1}{3}$

$= \frac{35}{6} = 5\frac{5}{6}$ (1)

$$\begin{array}{r} 5 \\ 6\overline{)35} \\ -30 \\ \hline 5 \end{array}$$

$n =$ $5\frac{5}{6}$

Question

A solid cuboid has a volume of 300 cm^3
The cuboid has a total surface area of 370 cm^2

The length of the cuboid is 20 cm.
The width of the cuboid is greater than the height of the cuboid.

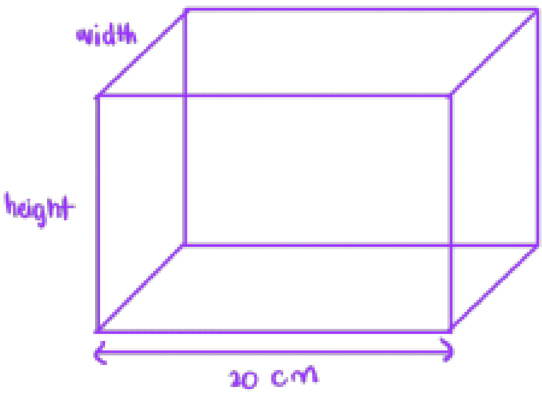
Work out the height of the cuboid.
You must show all your working.

MODEL ANSWER

Educate Cloud

The width of the cuboid is greater than the height of the cuboid.

Work out the height of the cuboid.
You must show all your working.



volume = length × width × height

$300\text{ cm}^3 = 20\text{ cm} \times \text{width} \times \text{height}$

$\text{width} \times \text{height} = \dfrac{300\text{ cm}^3}{20\text{ cm}} = 15\text{ cm}^2$ (1) —— ①

Total surface area = $2 \times (20 \times \text{height}) + 2 \times (20 \times \text{width}) + 2(\text{width} \times \text{height})$

$370\text{ cm}^2 = 2[(20 \times \text{height}) + (20 \times \text{width}) + (\text{width} \times \text{height})]$ (1)

$\dfrac{370\text{ cm}^2}{2} = (20 \times (\text{width} + \text{height})) + 15\text{ cm}^2$

$20(w + h) = 185 - 15$

$= 170$

$w + h = \dfrac{170}{20} = 8.5$

$w = 8.5 - h$ —— ②

② into ① ← eliminating w

$(8.5 - h)h = 15$ (1)

$h^2 - 8.5h + 15 = 0$

$(h - 2.5)(h - 6) = 0$ (1)

$h = 2.5$ or $h = 6$

since $w \times h = 15\text{ cm}^2$,

$h \neq 6$ because that would mean $h > w$.

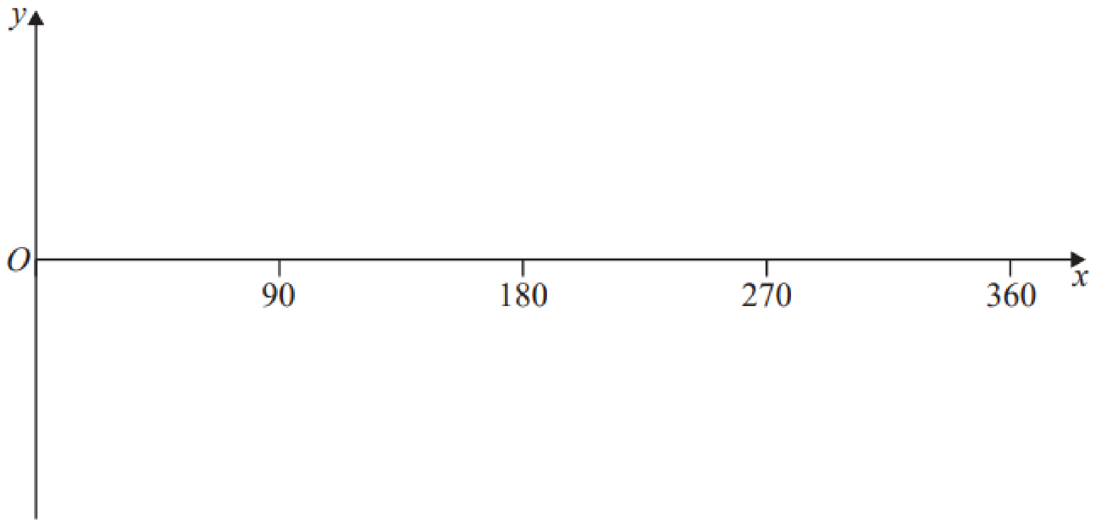
Hence, $h = 2.5$ (1)

2.5 .... cm

(Total for Question 21 is 5 marks)

Question

(a) Sketch the graph of $y = \sin x°$ for $0 \leqslant x \leqslant 360$



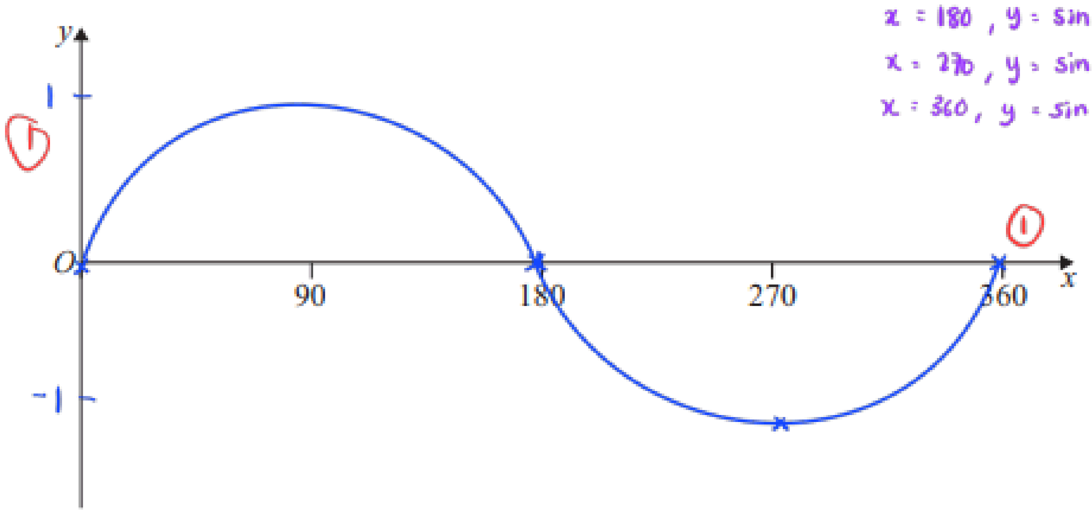
(2)

(b) Solve the equation $2\sin x° = -1$ for $0 \leqslant x \leqslant 360$

MODEL ANSWER

(a) Sketch the graph of $y = \sin x°$ for $0 \leqslant x \leqslant 360$

when $x = 0$, $y = \sin 0 = 0$
$x = 90$, $y = \sin 90° = 1$
$x = 180$, $y = \sin 180° = 0$
$x = 270$, $y = \sin 270° = -1$
$x = 360$, $y = \sin 360° = 0$



(2)

(b) Solve the equation $2\sin x° = -1$ for $0 \leqslant x \leqslant 360$

$2\sin x° = -1$

$\sin x° = -\frac{1}{2}$

since $\sin 30° = \frac{1}{2}$, (1)

for $-\frac{1}{2}$, x should be within $180°$ to $360°$ range

$x° = 180° + 30°$ and $360° - 30$

$= 210°$ and $330°$ (1)

$210°, 330°$

(2)

(Total for Question 22 is 4 marks)

Question

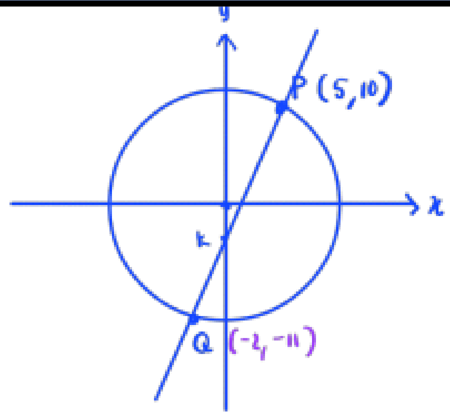
C is a circle with centre (0, 0)
L is a straight line.

The circle **C** and the line **L** intersect at the points P and Q.
The coordinates of P are (5, 10)
The x coordinate of Q is –2

L has a positive gradient and crosses the y-axis at the point $(0, k)$

Find the value of k.

MODEL ANSWER



$x^2 + y^2 = 5^2 + 10^2$

$= 25 + 100$ (1)

$= 125$

Finding coordinate of Q : $(-2)^2 + y^2 = 125$

$y^2 = 125 - 4$

$y^2 = 121$ (1)

$y = \pm\sqrt{121} = -11$ (since Q cannot be higher than P with positive gradient) (1)

$\therefore$ coordinate of Q : $(-2, -11)$

Gradient, m of line L : $\frac{10-(-11)}{5-(-2)} = \frac{21}{7} = 3$

Equation of line L : $10 = 3(5) + c$ (1) ← take point P

$10 = 15 + c$

$c = -5$

$k = -5$ (1)

$k = $ ..... -5

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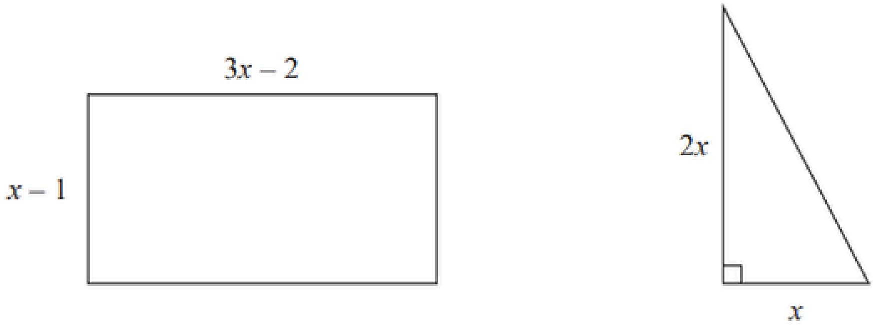
QUESTION

Here is a rectangle and a right-angled triangle.



All measurements are in centimetres.
The area of the rectangle is greater than the area of the triangle.

Find the set of possible values of x.

MODEL ANSWER

Find the set of possible values of x.

Area of rectangle

$= (3x-2)(x-1)$

$= 3x^2 - 3x - 2x + 2$

$= 3x^2 - 5x + 2$

Area of tri

$= \frac{1}{2} \times 2x \times x$

$= x^2$

$$3x^2 - 5x + 2 > x^2$$

$$2x^2 - 5x + 2 > 0$$

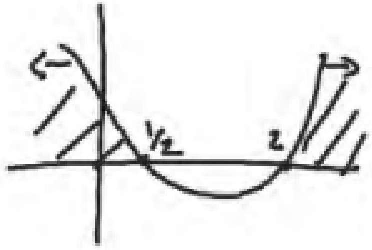
$2 \times 2 = 4$

$$2x^2 - 4x - x + 2 > 0$$

$$2x(x-2) - (x-2)$$

$$(2x-1)(x-2) > 0$$

$x < \frac{1}{2}$ $\quad$ $x > 2$



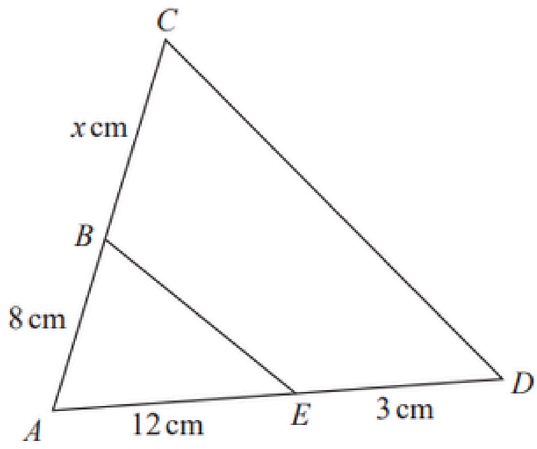
x can't be $< \frac{1}{2}$ as the width of the rectangle would be negative

$x > 2$

(Total for Question 23 is 5 marks)

QUESTION

The two triangles in the diagram are similar.



There are two possible values of x.

Work out each of these values.
State any assumptions you make in your working.

MODEL ANSWER

$$\frac{12}{3} = \frac{8}{x} \qquad \frac{8}{x} = 4 \qquad x = 2$$

$$8 = 4x$$

The triangles are similar so the lengths are in the same ratio

OR

$$\frac{x+8}{12} = \frac{12+3}{8}$$

$$\frac{x+8}{12} = \frac{15}{8}$$

$$8x + 64 = 180$$

$$8x = 116$$

$$15 \times 12: \quad 150 + 30 = 180$$

$$8 \overline{)116.0} = 14.5$$

$$x = 14.5\text{cm}$$

$x = 2$cm or 14.5cm

QUESTION

Show that $\dfrac{6-\sqrt{8}}{\sqrt{2}-1}$ can be written in the form $a+b\sqrt{2}$ where a and b are integers.

MODEL ANSWER

$$\frac{6-\sqrt{8}}{\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$$

$$\frac{6\sqrt{2}+6-\sqrt{16}-\sqrt{8}}{2+\sqrt{2}-\sqrt{2}-1} = \frac{6\sqrt{2}+6-4-2\sqrt{2}}{1} = 2+4\sqrt{2}$$

$$= 2+4\sqrt{2}$$

QUESTION

Educate Cloud

The table shows some values of x and y that satisfy the equation $y = a\cos x° + b$

x	0	30	60	90	120	150	180
y	3	$1 + \sqrt{3}$	2	1	0	$1 - \sqrt{3}$	-1

Find the value of y when $x = 45$

MODEL ANSWER

Find the value of y when $x = 45$

$\cos 0 = 1$

$3 = a\cos 0 + b$

$3 = a + b$

$1 = a\cos 90 + b$

$\cos 90 = 0$

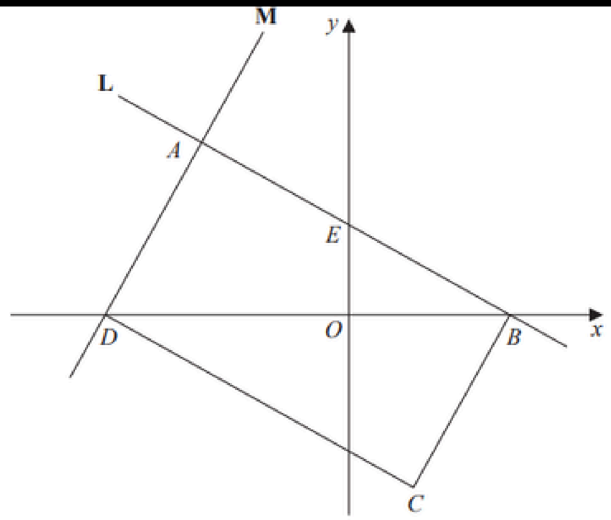
$b = 1$

$a = 2$

$y = 2\cos 45 + 1$

$y = 2 \times \frac{\sqrt{2}}{2} + 1 = \sqrt{2} + 1$

QUESTION



ABCD is a rectangle.

A, *E* and *B* are points on the straight line **L** with equation $x + 2y = 12$
A and *D* are points on the straight line **M**.

$AE = EB$

Find an equation for **M**.

MODEL ANSWER

ABCD is a rectangle.

A, *E* and *B* are points on the straight line **L** with equation $x + 2y = 12$
A and *D* are points on the straight line **M**.

$AE = EB$

Find an equation for **M**.

$2y = 12 - x$

$y = 6 - \frac{1}{2}x$

$E = x = 0 = (0, 6)$

$B = y = 0 = (12, 0)$

$A = (0 - 12, 6 + 6) = (-12, 12)$

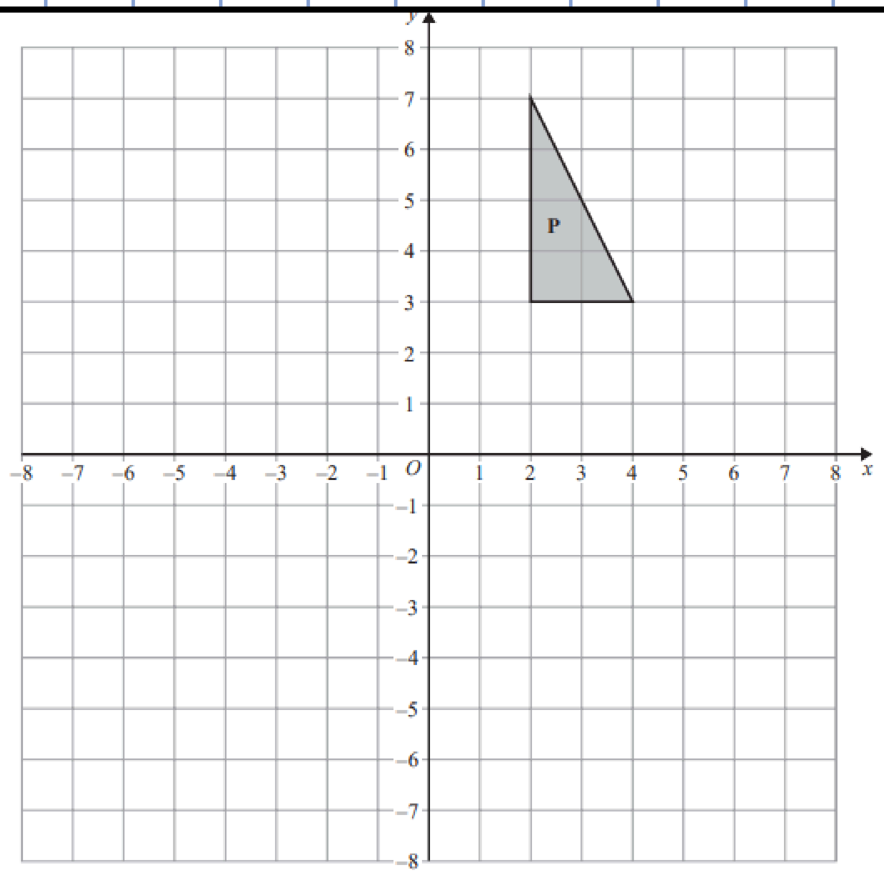
AD is perpendicular to AB
therefore gradient is neg recipricol of $-\frac{1}{2} = 2$

$m = 2 \quad x = -12 \quad y = 12$

$y = mx + c \quad 12 = -24 + c$

$12 = -12 \times 2 + c \quad c = 36 \quad y = 2x + 36$

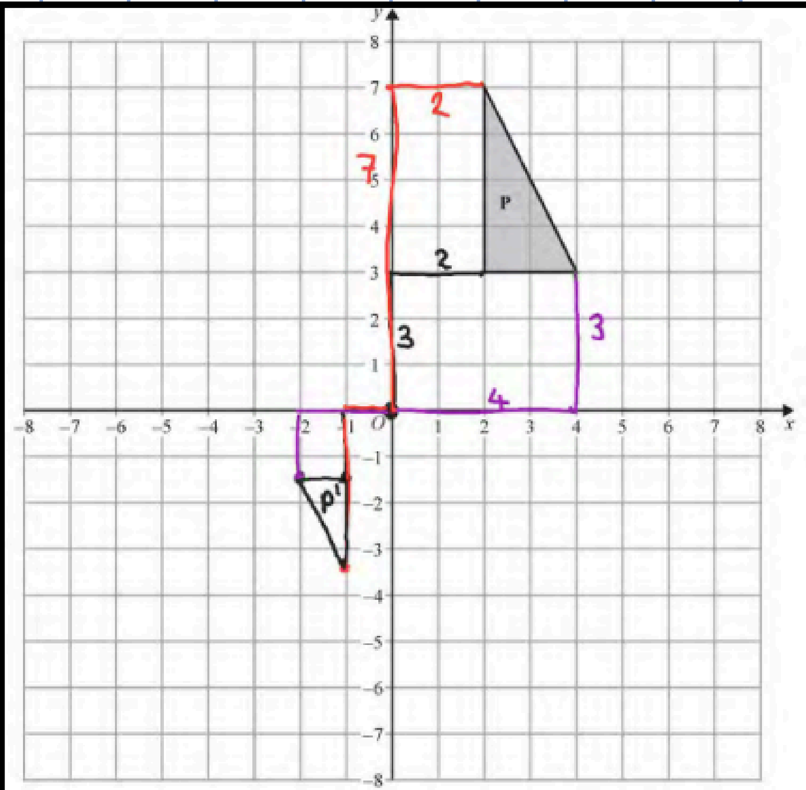
QUESTION



Enlarge shape **P** by scale factor $-\frac{1}{2}$ with centre of enlargement (0, 0).

Label your image **Q**.

MODEL ANSWER



Enlarge shape **P** by scale factor $-\frac{1}{2}$ with centre of enlargement (0, 0).

Label your image **Q**.

$\frac{1}{2}$ = shrink by 2

− = shape is enlarged in opp direction

(Total for Question 18 is 2 marks)

QUESTION

n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number.

MODEL ANSWER

$$\frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2) \leftarrow n^2+2n+n+2$$

$$= \frac{1}{2}(n^2+n + n^2+3n+2)$$

$$= \frac{1}{2}(2n^2 + 4n + 2)$$

$$= n^2 + 2n+1$$

$$= (n+1)^2$$

– therefore the answer is always a square number

QUESTION

Educate Cloud

y is directly proportional to $\sqrt[3]{x}$

$y = 1\frac{1}{6}$ when $x = 8$

Find the value of y when $x = 64$

MODEL ANSWER

$y \propto \sqrt[3]{x}$

$y = k\sqrt[3]{x}$

$1\frac{1}{6} = k\sqrt[3]{8}$

$\frac{7}{6} = 2k$

$\div 2$

$\frac{7}{12} = k$

$x = 64$

$y = \frac{7}{12} \times \sqrt[3]{64}$

$y = \frac{7}{12} \times 4 = \frac{28}{12} = \frac{7}{3}$

$y = \frac{7}{3}$

QUESTION

$x = 0.4\dot{3}\dot{6}$

Prove algebraically that x can be written as $\frac{24}{55}$

MODEL ANSWER

$x = 0.4363636$

$10x = 4.363636$

$100x = 43.636363$

$1000x = 436.3636$

$$\begin{aligned} 1000x &= 436.3636 \\ -\quad 10x &= 4.3636 \\ \hline 990x &= 432 \end{aligned}$$

$\therefore x = \dfrac{432}{990} = \dfrac{216}{495}$ ($\div 2$)

$\dfrac{216}{495} = \dfrac{24}{55}$ ($\div 9$)

QUESTION

The ratio $(y+x):(y-x)$ is equivalent to $k:1$

Show that $y = \dfrac{x(k+1)}{k-1}$

MODEL ANSWER

$$y+x : y-x = k : 1$$

$$\frac{y+x}{y-x} : 1 = k : 1$$

$$\therefore \frac{y+x}{y-x} = k$$

$$y+x = k(y-x)$$

$$y+x = ky - kx$$

$$kx + x = ky - y$$

$$x(k+1) = y(k-1)$$

$$y = \frac{x(k+1)}{k-1}$$

(Total fo

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QUESTION

There are only green pens and blue pens in a box.

There are three more blue pens than green pens in the box.
There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.

The probability that Simon will take two pens of the same colour is $\frac{27}{55}$

Work out the number of green pens in the box.

MODEL ANSWER

Let x be the number of green pens

green $= x$
blue $= x+3$
total $= 2x+3$

$P(\text{green}) = \dfrac{x}{2x+3}$

$P(\text{blue}) = \dfrac{x+3}{2x+3}$ ✓

$P(2\text{ green}) = \dfrac{x}{2x+3} \times \dfrac{x-1}{2x+2} = \dfrac{x(x-1)}{(2x+3)(2x+2)}$

$P(2\text{ blue}) = \dfrac{x+3}{2x+3} \times \dfrac{x+2}{2x+2} = \dfrac{(x+3)(x+2)}{(2x+3)(2x+2)}$ ✓

$P(2\text{ green}) + P(2\text{ blue}) = \dfrac{27}{55}$

$\dfrac{x(x-1)}{(2x+3)(2x+2)} + \dfrac{(x+3)(x+2)}{(2x+3)(2x+2)} = \dfrac{27}{55}$

$\dfrac{x(x-1)+(x+3)(x+2)}{(2x+3)(2x+2)} = \dfrac{27}{55}$

$\dfrac{2x^2+4x+6}{4x^2+10x+6} = \dfrac{27}{55}$

$\dfrac{x^2+2x+3}{2x^2+5x+3} = \dfrac{27}{55}$

$\times 55(2x^2+5x+3)$

$55(x^2+2x+3) = 27(2x^2+5x+3)$

$55x^2+110x+165 = 54x^2+135x+81$

$x^2-25x+84=0$ ✓

$(x-4)(x-21)=0$ ✓

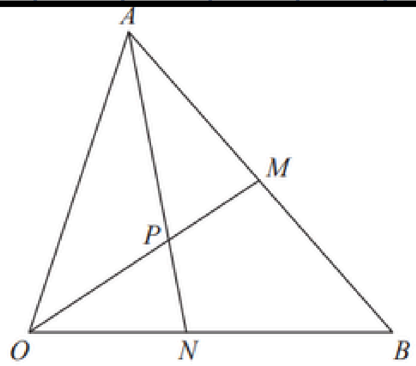
$x \geq 12$

$\therefore x = 21$

21 ✓

(Total for Question 22 is 6 marks)

QUESTION



OAB is a triangle.
OPM and *APN* are straight lines.
M is the midpoint of *AB*.

$\overrightarrow{OA} = \mathbf{a}$ $\quad \overrightarrow{OB} = \mathbf{b}$

$OP : PM = 3 : 2$

Work out the ratio $ON : NB$

MODEL ANSWER

Work out the ratio $ON : NB$

$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$

$= -a + b$ ✓

$\overrightarrow{AM} = \overrightarrow{MB} = \frac{1}{2}(-a+b)$

$= -\frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$= a - \frac{1}{2}a + \frac{1}{2}b$

$= \frac{1}{2}a + \frac{1}{2}b$ ✓

$\overrightarrow{OP} = \frac{3}{5}\overrightarrow{OM}$

$= \frac{3}{5}\left(\frac{1}{2}a + \frac{1}{2}b\right)$

$= \frac{3}{10}a + \frac{3}{10}b$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP}$

$= -a + \frac{3}{10}a + \frac{3}{10}b$

$= \frac{7}{10}a + \frac{3}{10}b$ ✓

$\overrightarrow{AN} = x\overrightarrow{AP}$

$= x\left(-\frac{7}{10}a + \frac{3}{10}b\right)$

$\overrightarrow{AN} = -\overrightarrow{OA} + \overrightarrow{ON}$

$= -a + 4b$

$-\frac{7}{10}xa + \frac{3}{10}xb = -a + 4b$

$-\frac{7}{10}x = -1$

$x = \frac{10}{7}$

$\frac{3}{10}x = 4$

$\frac{3}{10} \times \frac{10}{7} = 4 \qquad \frac{3}{7} = 4$ ✓

$\overrightarrow{ON} = \frac{3}{7}b$

$3 : 4$ ✓

QUESTION

Show that $\dfrac{(\sqrt{18}+\sqrt{2})^2}{\sqrt{8}-2}$ can be written in the form $a(b+\sqrt{2})$ where a and b are integers.

MODEL ANSWER

$$\frac{(\sqrt{18}+\sqrt{2})^2}{\sqrt{8}-2} \times \frac{\sqrt{8}+2}{\sqrt{8}+2}$$

$$= \frac{(\sqrt{18}+\sqrt{2})^2(\sqrt{8}+2)}{(\sqrt{8}-2)(\sqrt{8}+2)}$$

$$= \frac{(4\sqrt{2})^2(\sqrt{8}+2)}{8+2\sqrt{8}-2\sqrt{8}-4}$$

$$= \frac{32(\sqrt{8}+2)}{4}$$

$$= 8(\sqrt{8}+2)$$

$$= 8(2+2\sqrt{2})$$

$$= 16(1+\sqrt{2})$$

$$\sqrt{18} = \sqrt{2} \times \sqrt{9} = 3\sqrt{2}$$

$$(4\sqrt{2})^2 = 4^2 \times \sqrt{2}^2 = 16 \times 2 = 32$$

$$\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$$

QUESTION

For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(a) Show that $gf(x) = 2x(x + 2)$

(b) Find $g^{-1}(7)$

MODEL ANSWER

(a) Show that gf(x) = $2x(x + 2)$

$gf(x) = g(f(x))$

$f(x) = (x+1)^2$

$g(x) = ((x+1)^2) = 2((x+1)^2 - 1)$

$= 2(x^2 + 2x + 1 - 1)$

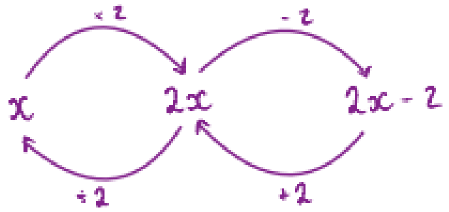
$= 2(x^2 + 2x)$

$= 2x(x+2)$

(b) Find $g^{-1}(7)$

$g(x) = 2(x-1)$

$= 2x - 2$



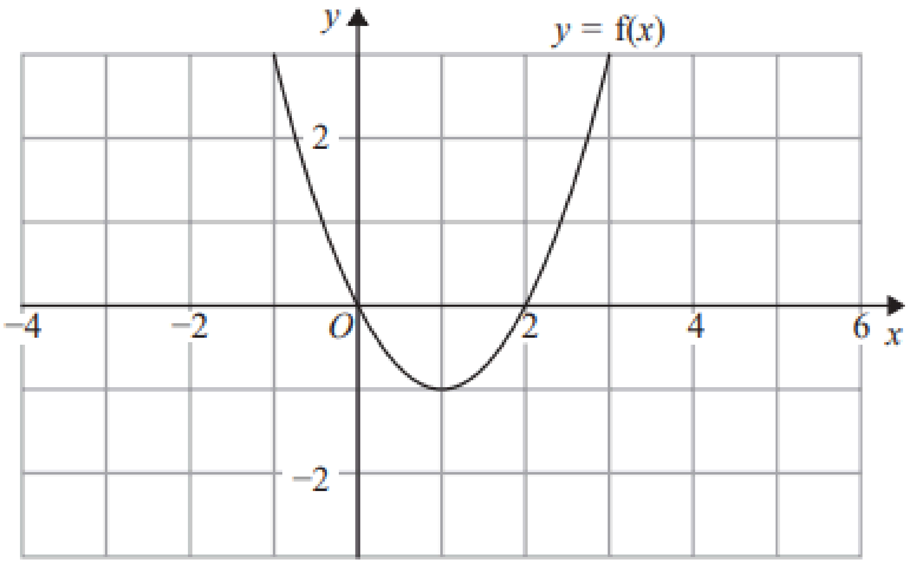
$g^{-1}(x) = \frac{x+2}{2}$

$g^{-1}(7) = \frac{(7)+2}{2}$

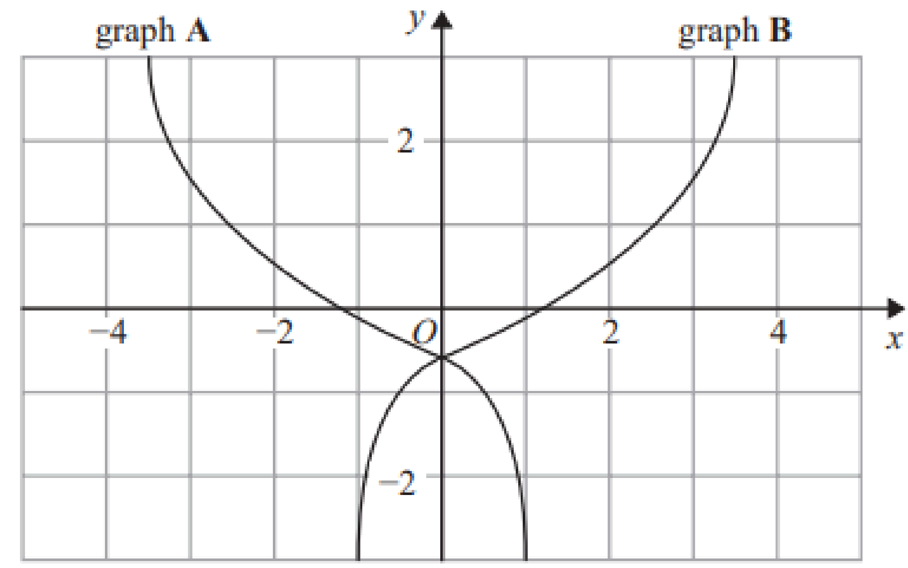
$= \frac{9}{2}$

QUESTION

The graph of $y = f(x)$ is shown on the grid below.



(a) On the grid above, sketch the graph of $y = f(x - 2)$



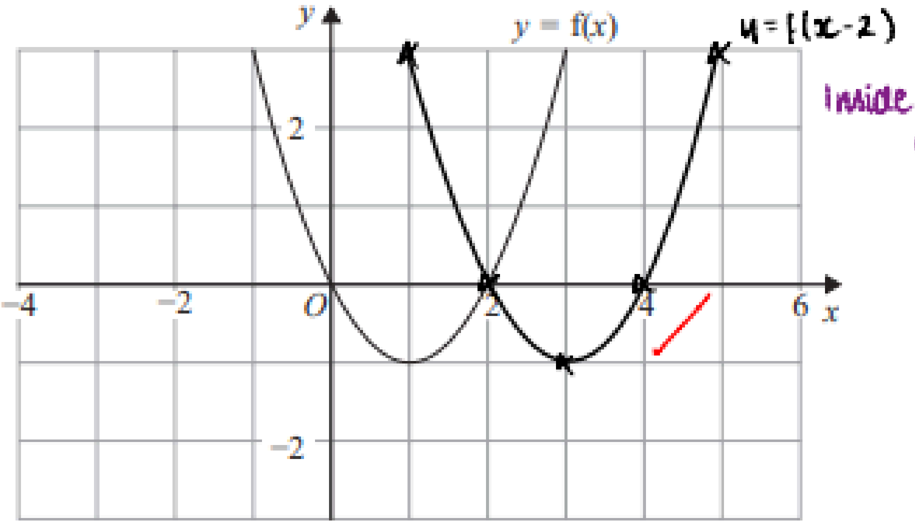
On the grid, graph **A** has been reflected to give graph **B**.

The equation of graph **A** is $y = g(x)$

(b) Write down the equation of graph **B**.

MODEL ANSWER

The graph of $y = f(x)$ is shown on the grid below.

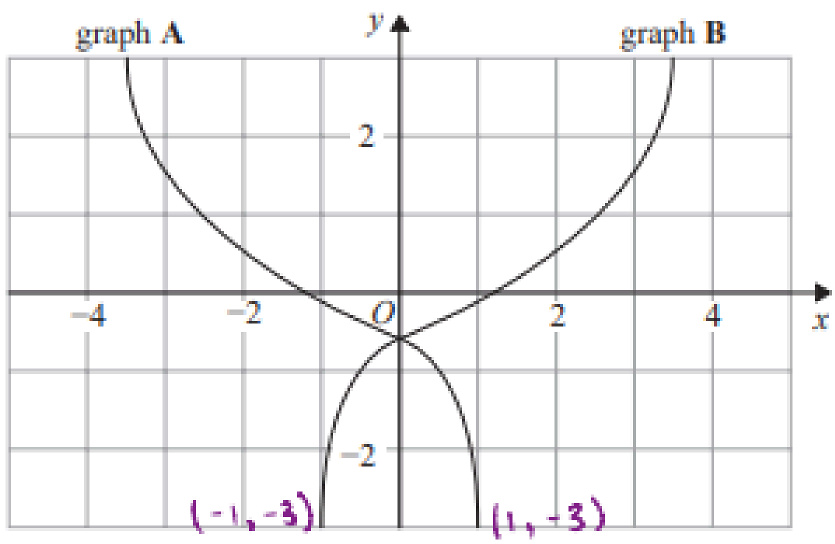


Inside brackets → do opposite of what is expected on x-axis

(a) On the grid above, sketch the graph of $y = f(x - 2)$

+2 all x coordinates

(1)



On the grid, graph **A** has been reflected to give graph **B**.

The equation of graph **A** is $y = g(x)$

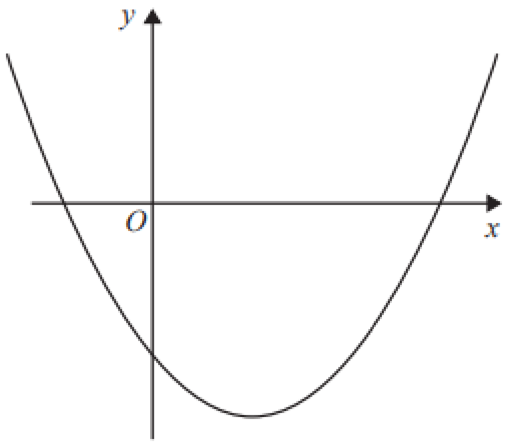
$y = g(-x)$

(b) Write down the equation of graph **B**.

$y = g(-x)$

QUESTION

Here is a sketch of a curve.



The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.

MODEL ANSWER

$x = 0$
$y = -5$

$-5 = (0)^2 + (0)a + b$
$-5 = b$ ✓

$x = 5$
$y = 0$
$b = -5$

$0 = (5)^2 + (5)a - 5$
$0 = 20 + 5a$
$-20 = 5a$
$-4 = a$ ✓

$y = x^2 - 4x - 5$
$y = (x-2)^2 - (-2)^2 - 5$
$y = (x-2)^2 - 4 - 5$
$y = (x-2)^2 - 9$ ✓

$y = \left(x + \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + b$
where $y = x^2 + ax + b$

$x = 2$
$y = 0^2 - 9$
$y = -9$
$(2, -9)$

(2 , -9) ✓

QUESTION

Educate Cloud

Prove algebraically that $0.2\dot{5}\dot{6}$ can be written as $\frac{127}{495}$

MODEL ANSWER

Let $x = 0.2\dot{5}\dot{6}$

$$x = 0.2565656\ldots$$
$$10x = 2.565656\ldots$$
$$100x = 25.656565\ldots$$
$$1000x = 256.5656\ldots$$

$$1000x - 10x = 990x$$
$$256.\dot{5}\dot{6} - 2.\dot{5}\dot{6} = 254$$

$$990x = 254$$
$(\div 990)$ $(\div 990)$
$$x = \frac{254}{990}$$
$$x = \frac{127}{495}$$

QUESTION

Educate Cloud

Three solid shapes **A**, **B** and **C** are similar.

The surface area of shape **A** is 4 cm^2
The surface area of shape **B** is 25 cm^2

The ratio of the volume of shape **B** to the volume of shape **C** is 27 : 64

Work out the ratio of the height of shape **A** to the height of shape **C**.
Give your answer in its simplest form.

MODEL ANSWER

A : B

SA 4 : 25

Lengths 2 : 5 ($\sqrt{\ }$)

↓×3↓

6 : 15

B : C

volume 27 : 64

Lengths 3 : 4 ($\sqrt[3]{\ }$)

↓×5↓

15 : 20

A : B : C

6 : 15 : 20

A : C

6 : 20

↓÷2↓

3 : 10

3 : 10

QUESTION

(a) Work out the value of $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

$3^a = \frac{1}{9}$ $\quad 3^b = 9\sqrt{3}$ $\quad 3^c = \frac{1}{\sqrt{3}}$

(b) Work out the value of $a + b + c$

MODEL ANSWER

(a) Work out the value of $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

$\frac{16}{81} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}$

$= \left(\left(\frac{16}{81}\right)^{\frac{1}{4}}\right)^{3}$

$= \left(\sqrt[4]{\frac{16}{81}}\right)^{3}$

$= \left(\frac{2}{3}\right)^{3} = \frac{2^{3}}{3^{3}} = \frac{8}{27}$

$\frac{8}{27}$

(2)

(b) Work out the value of $a + b + c$

$3^{a} = \frac{1}{9} = \frac{1}{3^{2}} = 3^{-2}$ $\quad a = -2$

$3^{b} = 9\sqrt{3} = 3^{2} \times 3^{\frac{1}{2}} = 3^{2.5}$ $\quad b = 2.5$

$3^{c} = \frac{1}{\sqrt{3}} = \frac{1}{3^{\frac{1}{2}}} = 3^{-\frac{1}{2}}$ $\quad c = -0.5$

$a + b + c$

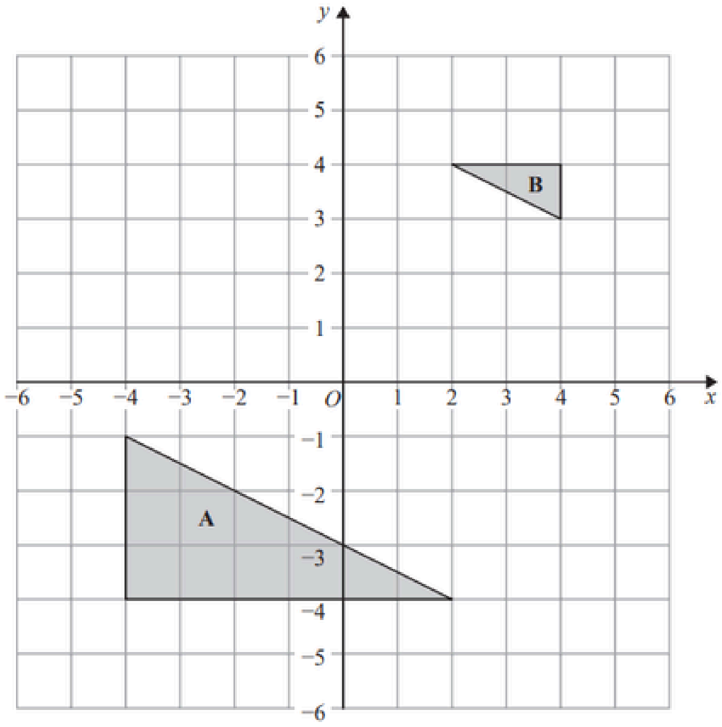
$= -2 + 2.5 - 0.5$

$= 0$

0

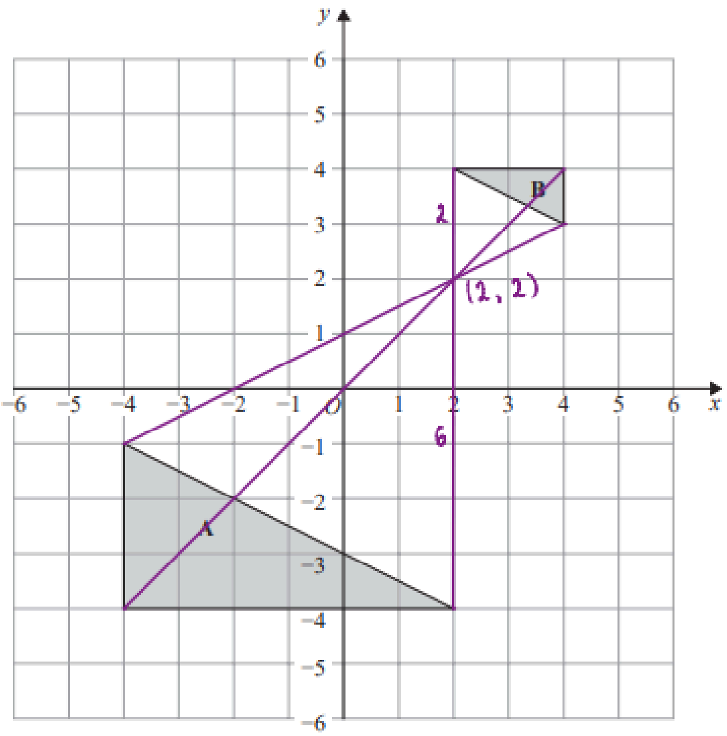
(2)

QUESTION



Describe fully the single transformation that maps triangle **A** onto triangle **B**.

MODEL ANSWER



Describe fully the single transformation that maps triangle **A** onto triangle **B**.

Enlargement scale factor $-\frac{1}{3}$ at centre (2, 2)

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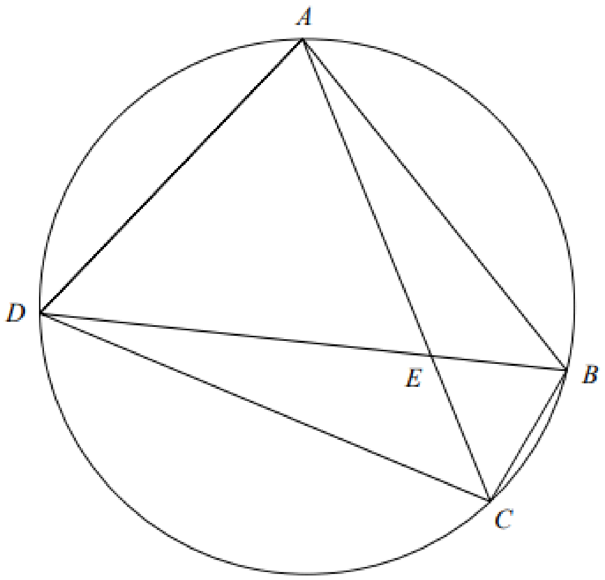
QUESTION

A, B, C and D are four points on a circle.

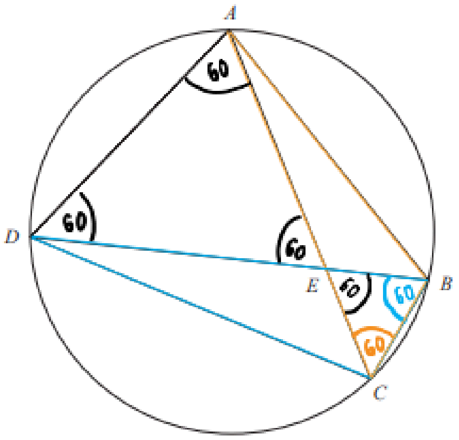


AEC and DEB are straight lines.

Triangle AED is an equilateral triangle.

Prove that triangle ABC is congruent to triangle DCB.

MODEL ANSWER



AEC and *DEB* are straight lines.

Triangle *AED* is an equilateral triangle. → SSS, ASA, SAS, RHS.

Prove that triangle *ABC* is congruent to triangle *DCB*.

Line BC is shared by both triangles. (1)

AED is equilateral ∴ ∠AED = ∠ADE = ∠DAE = 60° (1)

∠DAC = ∠DBC because angles in the same segment are equal.

∠ADB = ∠ACB because angles in the same segment are equal.

∴ ∠ACB = ∠DBC (1)

∠CEB = 60 ∴ ΔEBC is equilateral

AC = AE + EC = DE + EB = DB. ∴ AC = DB (1)

ΔABC is congruent to ΔDCB because they meet the SAS criteria.

QUESTION

Sketch the graph of

$$y = 2x^2 - 8x - 5$$

showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

Educate Cloud

MODEL ANSWER

Find y-intercept:

$y = ax^2 + bx + c$

c is always the y-intercept.

$y = 2x^2 - 8x - 5$

$c = -5$ ①

∴ y-intercept = -5

Find turning point: (complete the square)

$2x^2 - 8x - 5 = 0$

$2[x^2 - 4x] - 5 = 0$

$2[(x-2)^2 - 4] - 5 = 0$

$2(x-2)^2 - 8 - 5 = 0$

$2(x-2)^2 - 13 = 0$

$a(x+d)^2 + e = 0$

Turning point = $(-d, e)$

Turning point = $(2, -13)$ ①

Find x-intercepts:

$2(x-2)^2 - 13 = 0$

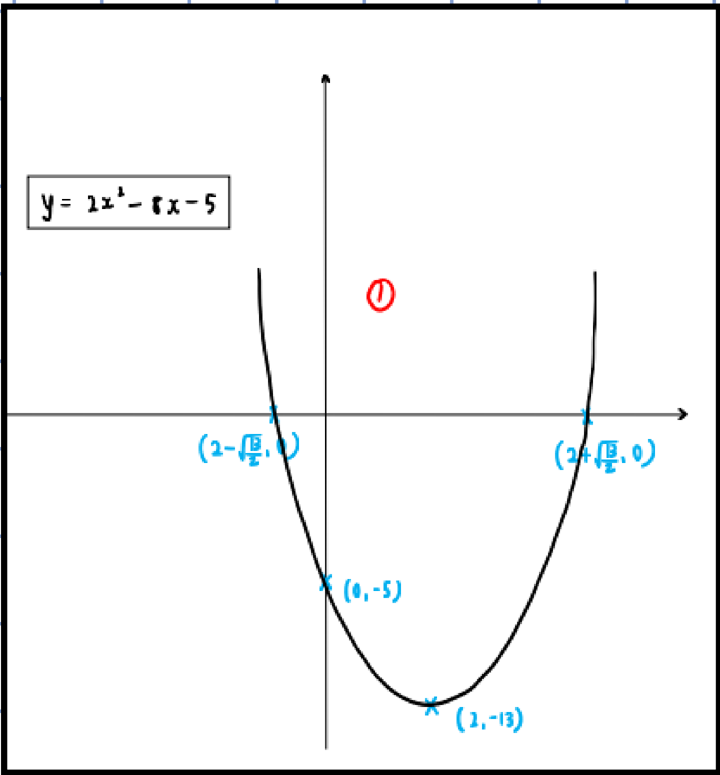
$2(x-2)^2 = 13$ ①

$(x-2)^2 = \frac{13}{2}$

$x - 2 = \pm\sqrt{\frac{13}{2}}$

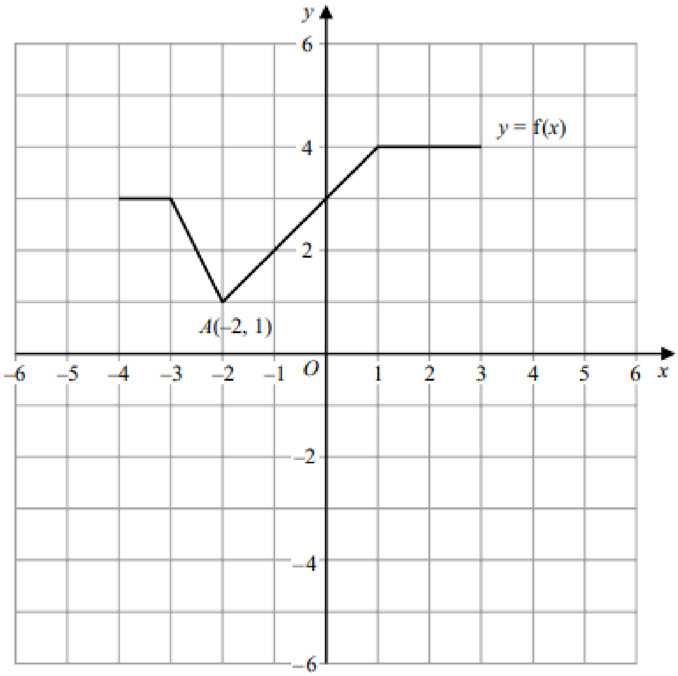
$x = 2 \pm \sqrt{\frac{13}{2}}$ ①

P.T.O.



QUESTION

The graph of $y = f(x)$ is shown on the grid.



(a) On the grid, draw the graph with equation $y = f(x + 1) - 3$

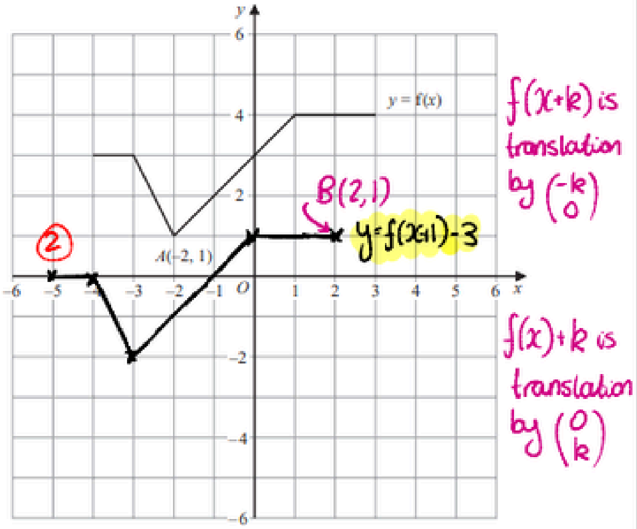
(2)

Point $A(-2, 1)$ lies on the graph of $y = f(x)$.

When the graph of $y = f(x)$ is transformed to the graph with equation $y = f(-x)$, point A is mapped to point B.

(b) Write down the coordinates of point B.

MODEL ANSWER



(a) On the grid, draw the graph with equation $y = f(x + 1) - 3$

translation by vector $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ (2)

Point $A(-2, 1)$ lies on the graph of $y = f(x)$.

When the graph of $y = f(x)$ is transformed to the graph with equation $y = f(-x)$, point A is mapped to point B.

(b) Write down the coordinates of point B.

$f(-x)$ is a reflection in the y axis

1

(2 , 1)

QUESTION

Given that $9^{-\frac{1}{2}} = 27^{\frac{1}{4}} \div 3^{x+1}$

find the exact value of x.

Educate Cloud

MODEL ANSWER

Given that $9^{-\frac{1}{2}} = 27^{\frac{1}{4}} \div 3^{x+1}$

find the exact value of x.

$$(3^2)^{-\frac{1}{2}} = \frac{(3^3)^{\frac{1}{4}}}{3^{x+1}} \quad (1)$$

$$3^{2\times-\frac{1}{2}} = \frac{3^{3\times\frac{1}{4}}}{3^{x+1}}$$

$$3^{-1} = \frac{3^{\frac{3}{4}}}{3^{x+1}} \quad (1)$$

$$3^{x+1} \times 3^{-1} = 3^{\frac{3}{4}}$$

$a^x \times a^y = a^{x+y}$
$(a^x)^y = a^{xy}$
$\frac{a^x}{a^y} = a^{x-y}$

$$3^{x+1-1} = 3^{\frac{3}{4}}$$

$$3^x = 3^{\frac{3}{4}}$$

$$\therefore x = \frac{3}{4} \quad (1)$$

$x = \frac{3}{4}$

QUESTION

The function f is given by

$$f(x) = 2x^3 - 4$$

(a) Show that $f^{-1}(50) = 3$

The functions g and h are given by

$$g(x) = x + 2 \quad \text{and} \quad h(x) = x^2$$

(b) Find the values of x for which

$$hg(x) = 3x^2 + x - 1$$

MODEL ANSWER

(a) Show that $f^{-1}(50) = 3$

$x = 2y^3 - 4$

$x + 4 = 2y^3$

$y^3 = \frac{x+4}{2}$

$y = \sqrt[3]{\frac{x+4}{2}}$

$f^{-1}(x) = \sqrt[3]{\frac{x+4}{2}}$ (1)

$\therefore f^{-1}(50) = \sqrt[3]{\frac{50+4}{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27} = 3$ (1)

(2)

The functions g and h are given by

(b) Find the values of x for which

$hg(x) = 3x^2 + x - 1$

$h(g(x)) = h(x+2) = (x+2)^2$

$\therefore hg(x) = (x+2)^2$ (1)

$(x+2)^2 = 3x^2 + x - 1$

$(x+2)(x+2)$ (1)

$x^2 + 4x + 4 = 3x^2 + x - 1$

$4x + 4 = 2x^2 + x - 1$

$4 = 2x^2 - 3x - 1$

$0 = 2x^2 - 3x - 5$

$2x^2 - 3x - 5 = 0$ (1)

$(2x - 5)(x + 1) = 0$

$2x - 5 = 0$, $x = 5/2$

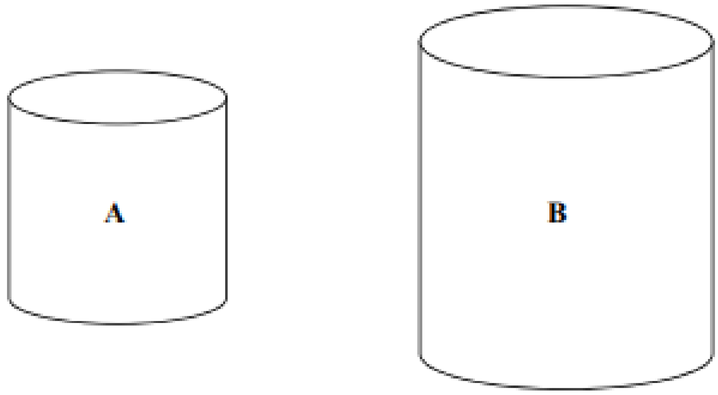
$x + 1 = 0$, $x = -1$

$x = 5/2$ and $x = -1$ (1)

(4)

QUESTION

A and **B** are two similar cylindrical containers.



the surface area of container **A** : the surface area of container **B** = 4 : 9

Tyler fills container **A** with water.
She then pours all the water into container **B**.
Tyler repeats this and stops when container **B** is full of water.

Work out the number of times that Tyler fills container **A** with water.
You must show all your working.

MODEL ANSWER

	A : B	
Surface area:	$4 : 9$	units^2
	$(\sqrt{4})$ $(\sqrt{9})$	
Length:	$2 : 3$	units (1)
	(2^3) (3^3)	
Volume:	$8 : 27$	units^3 (1)

(1) 8, 16, 24, 32

(1) (2) (3) (4)

$\therefore$ Tyler had to fill container A with water 4 times

(1) 4

QUESTION

(a) Rationalise the denominator of $\frac{22}{\sqrt{11}}$

Give your answer in its simplest form.

(b) Show that $\frac{\sqrt{3}}{2\sqrt{3}-1}$ can be written in the form $\frac{a+\sqrt{3}}{b}$ where a and b are integers.

MODEL ANSWER

(a) Rationalise the denominator of $\frac{22}{\sqrt{11}}$

Give your answer in its simplest form. (1)

$\sqrt{a} \times \sqrt{a} = a$

$$\frac{22}{\sqrt{11}} \frac{\times\sqrt{11}}{\times\sqrt{11}} = \frac{22\sqrt{11}}{\sqrt{11}\times\sqrt{11}} = \frac{22\sqrt{11}}{11} \begin{matrix}\div 11\\ \div 11\end{matrix} = \frac{2\sqrt{11}}{1} = 2\sqrt{11}$$

$2\sqrt{11}$ (1)

(2)

(b) Show that $\frac{\sqrt{3}}{2\sqrt{3}-1}$ can be written in the form $\frac{a+\sqrt{3}}{b}$ where a and b are integers.

$$\frac{\sqrt{3}}{2\sqrt{3}-1} \frac{\times(2\sqrt{3}+1)}{\times(2\sqrt{3}+1)} = \frac{\sqrt{3}(2\sqrt{3}+1)}{(2\sqrt{3}-1)(2\sqrt{3}+1)} = \frac{6+\sqrt{3}}{12+2\sqrt{3}-2\sqrt{3}-1} = \frac{6+\sqrt{3}}{11}$$

(1) (1) (1)

$a = 6 \quad b = 11$

QUESTION

Express $0.4\dot{1}\dot{8}$ as a fraction.
You must show all your working.

MODEL ANSWER

$x = 0.41818181818...$ (1)

$10x = 4.181818...$

$100x = 41.818181...$

$1000x = 418.181818...$

$1000x - 10x = 418.1818... - 4.1818...$

(1) $990x = 414.0$

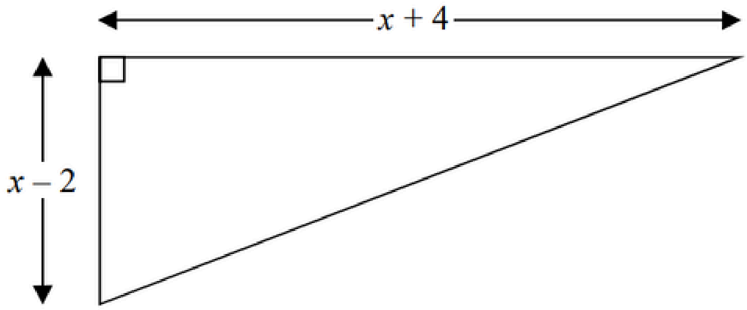
$x = \frac{414}{990}$

$0.4\dot{1}\dot{8} = \frac{414}{990}$

(1) $\frac{414}{990}$

QUESTION

The diagram shows a right-angled triangle.



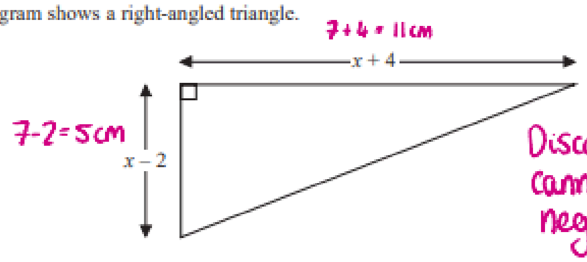
All the measurements are in centimetres.

The area of the triangle is $27.5\,cm^2$

Work out the length of the shortest side of the triangle.
You must show all your working.

MODEL ANSWER

The diagram shows a right-angled triangle.



$x = 7$

Discard $x = -9$ since cannot have negative length

All the measurements are in centimetres.

The area of the triangle is 27.5 cm²

Work out the length of the shortest side of the triangle.
You must show all your working.

$$\text{Area of Triangle} = \frac{\text{Base} \times \text{Height}}{2}$$

$$\text{Area} = \frac{1}{2} \times (x-2) \times (x+4) \quad (1)$$

$$2 \times \frac{1}{2}(x-2)(x+4) = 27.5 \times 2$$

$$(x-2)(x+4) = 55$$

$$x^2 + 4x - 2x - 8 = 55$$

$$x^2 + 2x - 8 = 55$$

$$-55 \quad -55$$

$$x^2 + 2x - 63 = 0 \quad (1)$$

$$-7 \times 9 = -63$$
$$-7 + 9 = 2$$

$$(x - 7)(x + 9) = 0$$

$$x - 7 = 0 \qquad x + 9 = 0 \quad (1)$$

$$x = 7 \text{ or } x = -9$$

(1) 5

QUESTION

(a) Write $\frac{5}{x+1} + \frac{2}{3x}$ as a single fraction in its simplest form.

(b) Factorise $(x+y)^2 + 3(x+y)$

MODEL ANSWER

(a) Write $\frac{5}{x+1} + \frac{2}{3x}$ as a single fraction in its simplest form.

$$\frac{5}{x+1} + \frac{2}{3x} = \frac{3x \times 5}{3x(x+1)} + \frac{2(x+1)}{3x(x+1)} \quad (1)$$

$$= \frac{15x}{3x(x+1)} + \frac{2x+2}{3x(x+1)} = \frac{15x+2x+2}{3x(x+1)} = \frac{17x+2}{3x(x+1)}$$

(1)

$\frac{17x+2}{3x(x+1)}$

(b) Factorise $(x + y)^2 + 3(x + y)$

$$(x+y)^2 + 3(x+y) = (x+y)[(x+y)+3]$$

(1)

$(x+y)(x+y+3)$

(1)

NOV 2020

QP and MA

HIGHER

QUESTION

There are four types of cards in a game.

Each card has a black circle or a white circle or a black triangle or a white triangle.

number of cards with a black shape	:	number of cards with a white shape	=	3 : 5
number of cards with a circle	:	number of cards with a triangle	=	2 : 7

Express the total number of cards with a black shape as a fraction of the total number of cards with a triangle.

MODEL ANSWER

Express the total number of cards with a black shape as a fraction of the total number of cards with a triangle.

↳ $\frac{\text{b.shape}}{\text{Triangle}} = \frac{3/8}{7/9} = \frac{3}{8} \times \frac{9}{7} = \frac{27}{56}$ ✓2

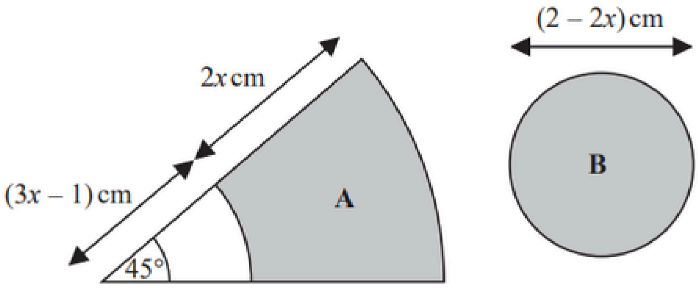
1/ fraction for b.shapes = $\frac{3}{8}$ ✓1 2/ fraction for triangle = 7/9

27/56 ✓3

QUESTION

The diagram shows two shaded shapes, **A** and **B**.

Shape **A** is formed by removing a sector of a circle with radius $(3x - 1)$ cm from a sector of the circle with radius $(5x - 1)$ cm.
Shape **B** is a circle of diameter $(2 - 2x)$ cm.

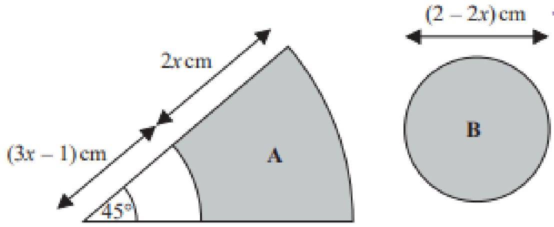


The area of shape **A** is equal to the area of shape **B**.

Find the value of x.
You must show all your working.

MODEL ANSWER

Shape **A** is formed by removing a sector of a circle with radius $(3x - 1)$ cm from a sector of the circle with radius $(5x - 1)$ cm.
Shape **B** is a circle of diameter $(2 - 2x)$ cm.



$\rightarrow r = \frac{1}{2}(2-2x)$

$r = 1 - x$

The area of shape **A** is equal to the area of shape **B**.

$a.s = \frac{\theta}{360} \times \pi r^2$

Find the value of x.
You must show all your working.

Area of shape A = area of sector - cutout.

area of sector $= \frac{45°}{360°} \times \pi \times (5x-1)^2 \text{ cm}^2$

$= \frac{1}{8} \times \pi \times (5x-1)^2$

cutout $= \frac{45°}{360°} \times \pi \times (3x-1)^2 \text{ cm}^2$

$= \frac{1}{8} \times \pi (3x-1)^2$

$(5x-1)(5x-1)$
$25x^2 - 5x - 5x + 1$

$= \frac{1}{8}\pi\left((5x-1)^2 - (3x-1)^2\right)$ ✓1

$(3x-1)(3x-1)$
$9x^2 - 3x - 3x + 1$

$= \frac{1}{8}\pi\left((25x^2 - 10x + 1) - (9x^2 - 6x + 1)\right)$

$= \frac{1}{8}\pi(16x^2 - 4x)$ ✓2

$(1-x)(1-x)$
$1 - x - x + x^2$

area of B $= \pi(1-x)^2 = \pi(x^2 - 2x + 1)$

$\times 8$: $\frac{1}{8}(16x^2 - 4x)\pi = \pi(x^2 - 2x + 1)$ $\times 8$

$16x^2 - 4x = 8x^2 - 16x + 8$ $-(8x^2 - 16x + 8)$

$8x^2 + 12x - 8 = 0$

$2x^2 + 3x - 2 = 0$ ✓3

$x_1, x_2 = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2 \times 2}$

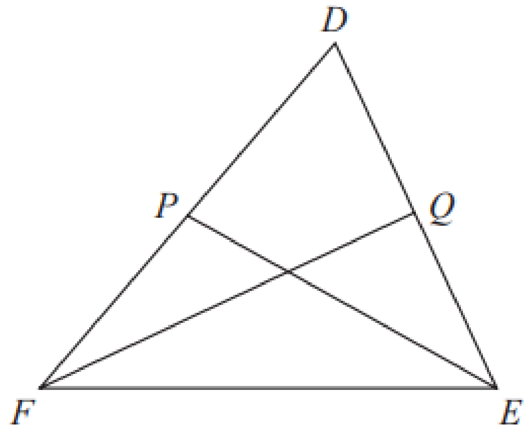
$= \frac{1}{2}, -2$ ✓4

$x = \frac{1}{2}$ ✓5

(Total for Question 22 is 5 marks)

QUESTION

DEF is a triangle.

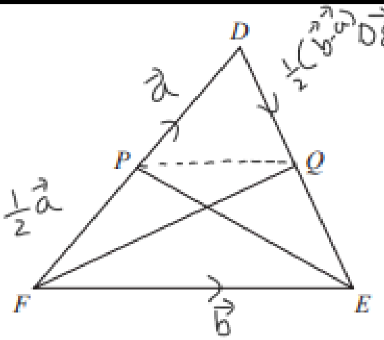


P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

MODEL ANSWER



P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

$\overrightarrow{DE} = \overrightarrow{DF} + \overrightarrow{FE}$
$= -\vec{a} + \vec{b}$
$= \vec{b} - \vec{a}$

$\overrightarrow{DQ} = \frac{1}{2}\overrightarrow{DE}$
$= \frac{1}{2}(\vec{b} - \vec{a})$ ✓1

$\overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DQ}$
$= \frac{1}{2}\vec{a} + \frac{1}{2}(\vec{b} - \vec{a})$ ✓2
$= \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$
$= \frac{1}{2}\vec{b}$ ✓3

for 2 vectors to be parallel, they must be scalar multiples of eachother.

$\overrightarrow{PQ} = \frac{1}{2}\vec{b}$ and $\overrightarrow{FE} = \vec{b}$

$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{FE}$ and so $\overrightarrow{PQ}$ is a scalar multiple of $\overrightarrow{FE}$.

Therefore, $\overrightarrow{PQ}$ and $\overrightarrow{FE}$ are parallel, as required. ✓4

QUESTION

Show that $\dfrac{\sqrt{180} - 2\sqrt{5}}{5\sqrt{5} - 5}$ can be written in the form $a + \dfrac{\sqrt{5}}{b}$ where a and b are integers.

MODEL ANSWER

$$\sqrt{180} = \sqrt{9 \times 20}$$
$$= \sqrt{9} \times \sqrt{20}$$
$$= 3 \times \sqrt{20}$$
$$= 3 \times \sqrt{4 \times 5}$$
$$= 3 \times \sqrt{4} \times \sqrt{5}$$
$$= 3 \times 2 \times \sqrt{5} = 6\sqrt{5} \checkmark_1$$

$$\frac{a}{b-c} = \frac{a(b+c)}{(b-c)(b+c)} \quad b^2 - c^2$$

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$$

$$\frac{6\sqrt{5} - 2\sqrt{5}}{5\sqrt{5} - 5} = \frac{4\sqrt{5}}{5\sqrt{5} - 5}$$

$$= \frac{4\sqrt{5}(5\sqrt{5} + 5)}{(5\sqrt{5} - 5)(5\sqrt{5} + 5)} \checkmark_2$$

$$= \frac{100 + 20\sqrt{5}}{125 - 25}$$

$$= \frac{100 + 20\sqrt{5}}{100} \checkmark_3$$

$$= \frac{100}{100} + \frac{20\sqrt{5}}{100}$$

$$= 1 + \frac{\sqrt{5}}{5} \checkmark_4$$

$\therefore a = 1$

$b = 5$

QUESTION

f and g are functions such that

$$f(x) = \frac{12}{\sqrt{x}} \quad \text{and} \quad g(x) = 3(2x + 1)$$

(a) Find g(5)

(b) Find gf(9)

(c) Find $g^{-1}(6)$

MODEL ANSWER

(a) Find g(5)

↳ Substitute 5 for x in g

$g(5) = 3(2 \times 5 + 1)$
$= 3(11) = 33$

33 ✓1

(1)

(b) Find gf(9)

$f(x) = \frac{12}{\sqrt{x}}$ $g(x) = 3(2x+1)$

$g(f(9))$

$f(9) = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$ ✓1

$g(f(9)) = g(4) = 3(2 \times 4 + 1)$
$= 27$

27 ✓2

(2)

(c) Find $g^{-1}(6)$ $g(x) = 3(2x+1)$

① finding $g^{-1}(x)$

→ $x = 3(2y+1)$ (rearrange for y)

÷3: $\frac{x}{3} = 2y + 1$

−1: $\frac{x}{3} - 1 = 2y$

÷2: $y = \frac{1}{2}\left(\frac{x}{3} - 1\right)$

$\therefore g^{-1}(x) = \frac{1}{2}\left(\frac{x}{3} - 1\right)$ ✓1

→ $g^{-1}(6) = \frac{1}{2}\left(\frac{6}{3} - 1\right)$
$= \frac{1}{2} \times 1 = \frac{1}{2}$

½ ✓2

(2)

(Total for Question 19 is 5 marks)

QUESTION

x is proportional to $\sqrt{y}$ where $y > 0$

y is increased by 44%

Work out the percentage increase in x.

MODEL ANSWER

Work out the percentage increase in x.

$\hookrightarrow x_n = K \times \sqrt{y \times 1.44}$ ✓2 $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

$x_n = K \times \sqrt{y} \times \sqrt{1.44}$

$x_n = K\sqrt{y} \times 1.2$

$x_n = x \times 1.2 \rightarrow 20\%$ increase ✓3

QUESTION

Make f the subject of the formula $d = \dfrac{3(1 - f)}{f - 4}$

MODEL ANSWER

Make f the subject of the formula $d = \frac{3(1-f)}{f-4}$

$\times(f-4)$

$$(f-4)d = 3(1-f)$$

$$fd - 4d = 3 - 3f$$

$$fd + 3f = 4d + 3$$

$\div(d+3)$

$$f(d+3) = 4d + 3$$

$$f = \frac{4d+3}{d+3}$$

$$f = \frac{4d+3}{d+3}$$

QUESTION

Shirley wants to find an estimate for the number of bees in her hive.

On Monday she catches 90 of the bees.
She puts a mark on each bee and returns them to her hive.

On Tuesday she catches 120 of the bees.
She finds that 20 of these bees have been marked.

(a) Work out an estimate for the total number of bees in her hive.

Shirley assumes that none of the marks had rubbed off between Monday and Tuesday.

(b) If Shirley's assumption is wrong, explain what effect this would have on your answer to part (a).

MODEL ANSWER

(a) Work out an estimate for the total number of bees in her hive.

M : T
20 : 120

m : T
90 : n

$$\frac{20}{120} = \frac{90}{n}$$ ✓1,2

$\times n$

$$\frac{20n}{120} = 90$$

$\div 20$

$$20n = 90 \times 120$$

$$n = \frac{90 \times 120}{20}$$

$\div 20$

$$n = 90 \times 6 = 540$$ ✓3

540

(b) If Shirley's assumption is wrong, explain what effect this would have on your answer to part (a).

fewer marked bees, This means the answer will be over-estimated. ✓

QUESTION

The straight line L_1 has equation $y = 3x - 4$
The straight line L_2 is perpendicular to L_1 and passes through the point (9, 5)

Find an equation of line L_2

MODEL ANSWER

$m_{L1} \times m_{L2} = -1$

$3 \times m_{L2} = -1$

$m_{L2} = -\frac{1}{3}$ ✓1

$y = -\frac{1}{3}x + c$

at $x = 9$, $y = 5$

$5 = -\frac{1}{3} \times 9 + c$ ✓2

$5 = -3 + c$

$5 = -3 + c$ $\downarrow +3$

$c = 8$

$y = -\frac{1}{3}x + 8$ ✓3

QUESTION

Sally plays two games against Martin.
In each game, Sally could win, draw or lose.

In each game they play,

the probability that Sally will win against Martin is 0.3

the probability that Sally will draw against Martin is 0.1

Work out the probability that Sally will win **exactly** one of the two games against Martin.

MODEL ANSWER

In each game they play,

the probability that Sally will win against Martin is 0.3

the probability that Sally will draw against Martin is 0.1

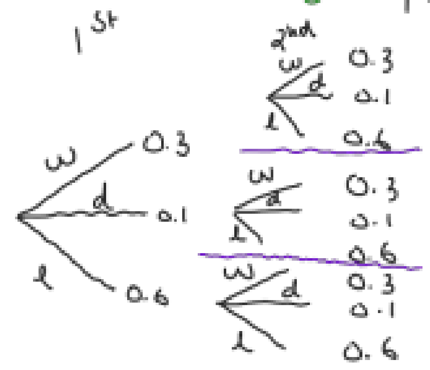
$P(l) = 0.6$

Work out the probability that Sally will win **exactly** one of the two games against Martin.

$P(w) + P(d) + P(l) = 1$

$0.3 + 0.1 + p(l) = 1 \rightarrow p(l) + 0.4 = 1$) -0.4

0.4 $p(l) = 0.6$



0.03
0.18
0.03
0.18
0.42
2

$P(\text{exactly } 1) = 0.3 \times 0.1 + 0.3 \times 0.6$ ✓1

$+ 0.1 \times 0.3 + 0.6 \times 0.3$

$= 0.03 + 0.18 + 0.03 + 0.18$ ✓2

$= 0.42$

0.42 ✓3

NOV 2021

QP and MA

HIGHER

QUESTION

Find the coordinates of the turning point on the curve with equation $y = 9 + 18x - 3x^2$
You must show all your working.

MODEL ANSWER

Educate Cloud

$y = -3x^2 + 18x + 9.$

Factorise the -3:

$y = -3(x^2 - 6x) + 9.$ (1)

We know that $(x^2 - 2ax) = (x-a)^2 - a^2$

$\therefore y = -3[(x-3)^2 - 9] + 9.$ (1)

Multiply by -3:

$y = -3(x-3)^2 + 27 + 9.$

$y = -3(x-3)^2 + 36.$ (1)

If $y = (x-a)^2 + b$, T.P is (a, b)

$\therefore$ turning point $= (3, 36)$.

(1)

(3 , 36)

(Total for Question 22 is **4 marks**)

QUESTION

The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{for } x > 0 \qquad \text{and} \qquad g(x) = \frac{4}{x^2} \quad \text{for } x > 0$$

(a) Work out gf(1)

The function h is such that $h = (fg)^{-1}$

(b) Find $h(x)$

MODEL ANSWER

Educate Cloud

(a) Work out gf(1) $g(f(1))$.

Start with f(1): $f(1) = 3(1)^2 + 1 = 3 + 1 = 4$. (1)

$f(1) = 4 \therefore g(f(1)) = g(4) = \frac{4}{4^2} = \frac{4}{16} = \frac{1}{4}$

(1) $\frac{1}{4}$

(b) Find h(x)

$(fg)^{-1}$ is the inverse of $f(g)$.

Find f(g): $f(x) = 3x^2 + 1$.

$f\left(\frac{4}{x^2}\right) = 3\left(\frac{4}{x^2}\right)^2 + 1$. (1)

$= 3\left(\frac{16}{x^4}\right) + 1 = \frac{48}{x^4} + 1$.

$f(g) = \frac{48}{x^4} + 1$. Find inverse:

Let $y = \frac{48}{x^4} + 1$. Make x the subject (1)

$y - 1 = \frac{48}{x^4}$

$\times x^4$

$x^4(y-1) = 48$.

$\div(y-1)$

$x^4 = \frac{48}{y-1}$

(1) $x = \sqrt[4]{\frac{48}{y-1}}$

Now swap the y with an x, and swap the x with $(fg)^{-1}$:

$(fg)^{-1} = \sqrt[4]{\frac{48}{x-1}}$

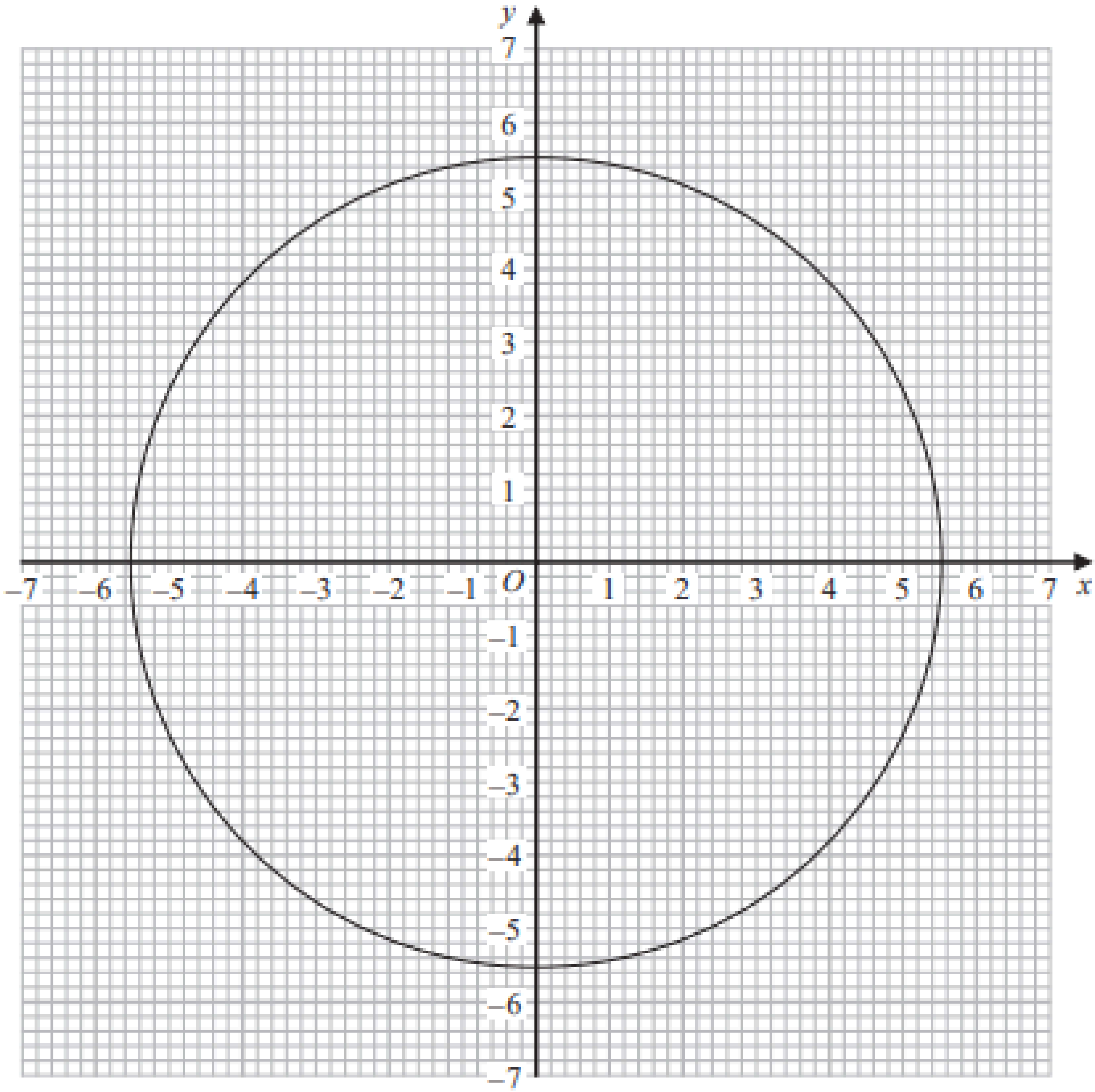
(1) $\sqrt[4]{\frac{48}{x-1}}$

(4)

(Total for Question 21 is 6 marks)

QUESTION

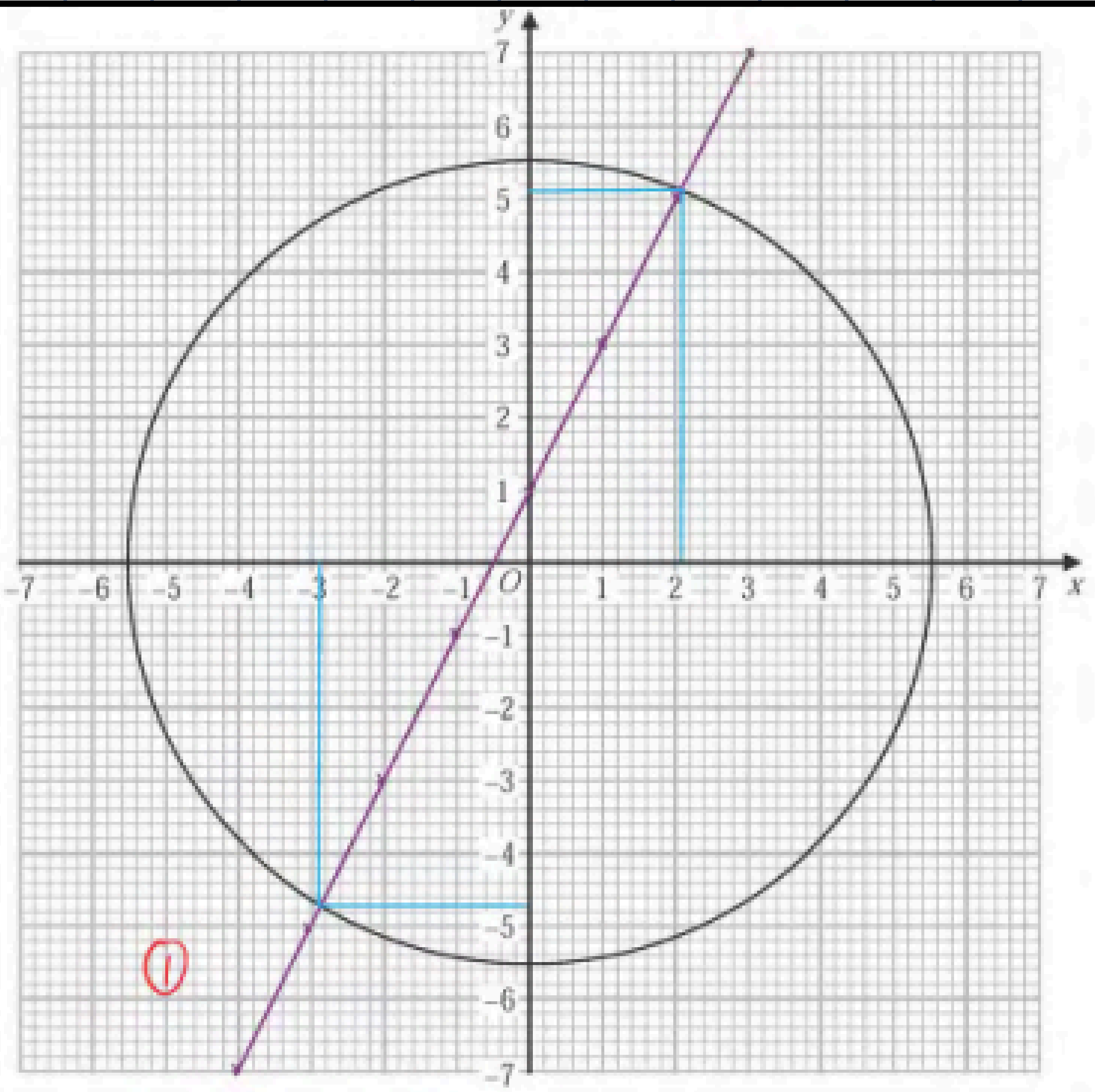
The diagram shows the graph of $x^2 + y^2 = 30.25$



Use the graph to find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 30.25$$
$$y - 2x = 1$$

MODEL ANSWER



Use the graph to find estimates for the solutions of the simultaneous equations

Point at which the circle and line intersect.

$x^2 + y^2 = 30.25$

$y - 2x = 1$ ∴ $y = 2x + 1$

Gradient = $\frac{\text{change in } y}{\text{change in } x}$ = 2.

$2 = \frac{2}{1}$ ∴ $\Delta y = 2$, $\Delta x = 1$.

Δ is the Greek letter Delta, which, in maths, generally means 'change in'.

(1) (1)

(2.1, 5.1) and (−2.9, −4.7).

(Total for Question 20 is 3 marks)

QUESTION

Educate Cloud

Show that $\frac{8+\sqrt{12}}{5+\sqrt{3}}$ can be written in the form $\frac{a+\sqrt{3}}{b}$, where a and b are integers.

MODEL ANSWER

Educate Cloud

Rationalise the denominator using 'Difference of two squares.'

$$\frac{8+\sqrt{12}}{5+\sqrt{3}} \quad \begin{matrix} \times (5-\sqrt{3}) \\ \times (5-\sqrt{3}) \end{matrix} \quad (1)$$

$\sqrt{a} \times \sqrt{a} = a$

$$= \frac{(8+\sqrt{12})(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} \quad (1)$$

$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

$\therefore 5\sqrt{12} = 5 \times 2\sqrt{3} = 10\sqrt{3}$

$$= \frac{40 - 8\sqrt{3} + 5\sqrt{12} - (\sqrt{3})(\sqrt{12})}{25 - 5\sqrt{3} + 5\sqrt{3} - (\sqrt{3})(\sqrt{3})}$$

$$= \frac{40 - 8\sqrt{3} + 10\sqrt{3} - \sqrt{36}}{25 - 3}$$

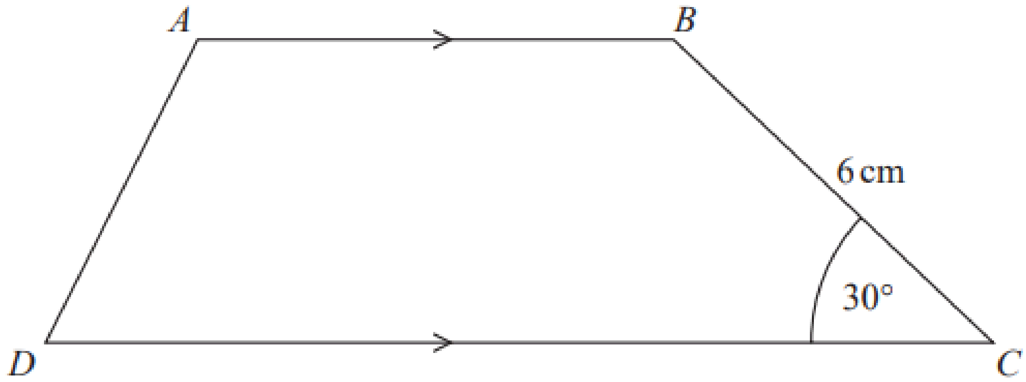
$$= \frac{40 + 2\sqrt{3} - 6}{22} = \frac{34 + 2\sqrt{3}}{22} \quad (1)$$

$$= \boxed{\frac{17 + \sqrt{3}}{11}} \quad (1)$$

(Total for Question 19 is **4 marks**)

QUESTION

Here is trapezium *ABCD*.



The area of the trapezium is $66\,\text{cm}^2$

the length of AB : the length of $CD = 2:3$

Find the length of AB.

MODEL ANSWER

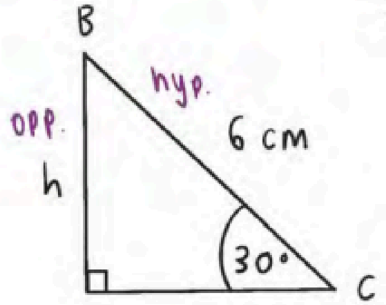
Educate Cloud

Find the length of *AB*.

AB : CD
= 2 : 3

5 parts in total.
AB has 2 of these 5 parts.
CD has 3 of these 5 parts.

Find height of trapezium:



$\sin 30 = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{6}$

$\sin 30 = \frac{1}{2}$ (1)

$\frac{1}{2} = \frac{h}{6}$ (1)

×6 ×6

$3 = h$

Area of trapezium: (1)

$A = \left(\frac{a+b}{2}\right)h$ $66 = \left(\frac{\frac{2}{5}x + \frac{3}{5}x}{2}\right)(3)$

Find length AB: (1)   (1)

$66 = \left(\frac{x}{2}\right)(3)$

÷3 ÷3

$22 = \frac{x}{2}$

×2 ×2

$44 = x$

$AB = \frac{2}{5}x$

$= \frac{2}{5}(44)$

$= 17.6 \text{ cm}$

17.6 .... cm

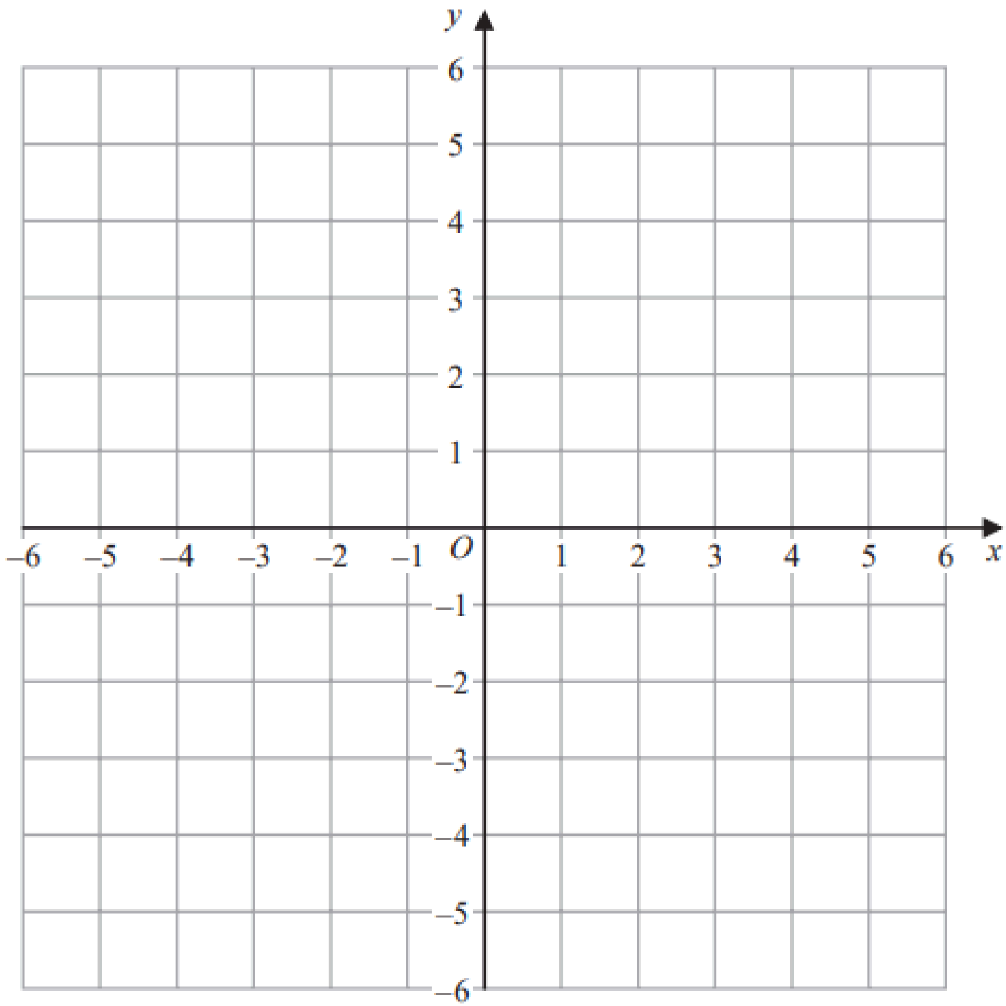
(Total for Question 18 is **5 marks**)

QUESTION

On the grid show, by shading, the region that satisfies all of these inequalities.

$2y + 4 < x \qquad x < 3 \qquad y < 6 - 3x$

Label the region **R**.



MODEL ANSWER

Educate Cloud

On the grid show, by shading, the region that satisfies all of these inequalities.

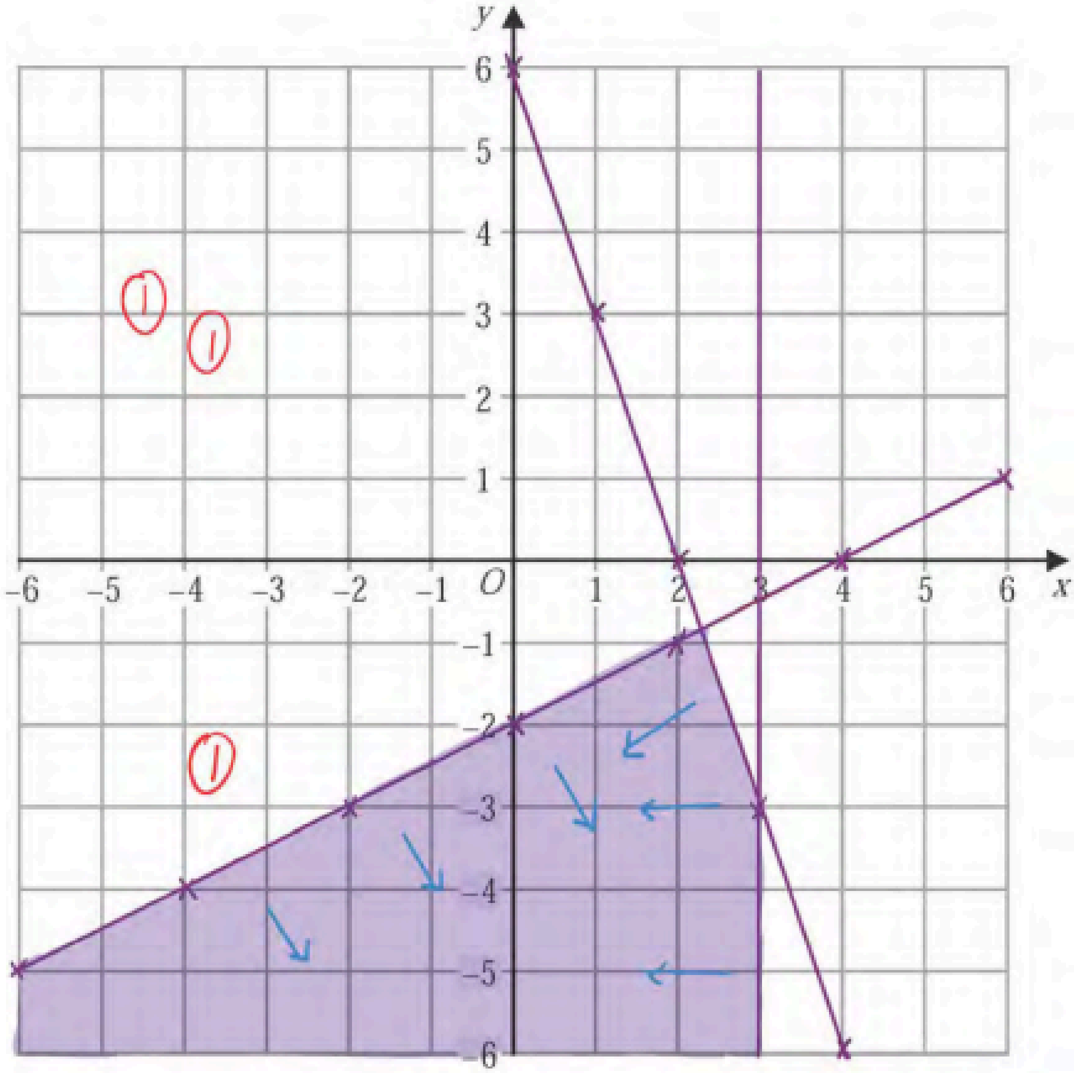
$2y + 4 < x$ $\quad$ $x < 3$ $\quad$ $y < 6 - 3x$

Label the region **R**.

$2y < x - 4$

$y < \frac{1}{2}x - 2$

$y < -3x + 6$



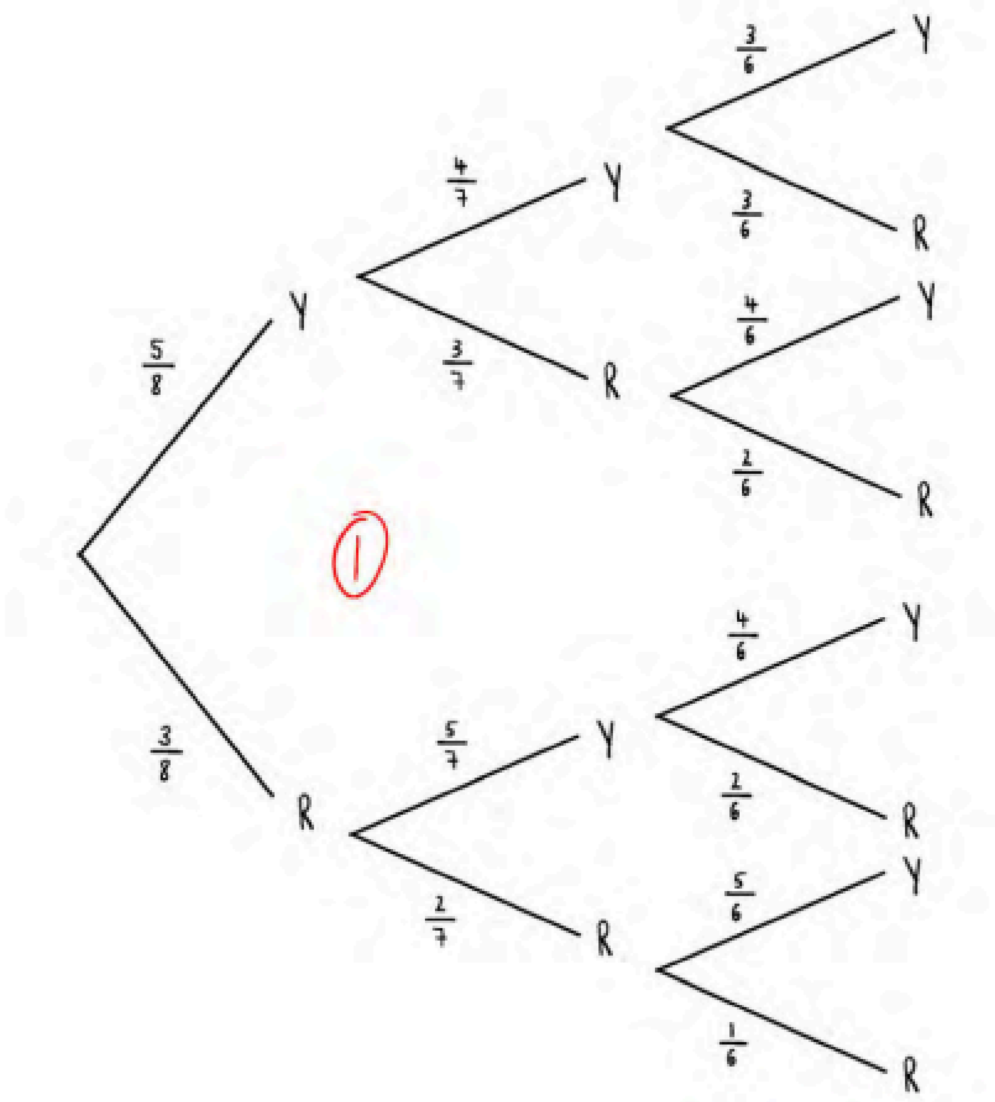
QUESTION

There are only 3 red counters and 5 yellow counters in a bag.

Jude takes at random 3 counters from the bag.

Work out the probability that he takes exactly one red counter.

MODEL ANSWER



P(exactly one Red) = P(RYY) OR P(YRY) OR P(YYR)

$$= \left(\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6}\right) + \left(\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6}\right) + \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right)$$ (1)

$$= \frac{60}{336} + \frac{60}{336} + \frac{60}{336} = \boxed{\frac{180}{336}}$$

(1) (1)

$\frac{180}{336}$

QUESTION

Show that $\frac{4x+3}{2x} + \frac{3}{5}$ can be written in the form $\frac{ax+b}{cx}$ where a, b and c are integers.

MODEL ANSWER

Make both fractions have a common denominator of $10x$:

$$\frac{4x+3}{2x} \xrightarrow{\times 5} \frac{(5)\,4x+3}{(5)\,2x} = \frac{20x+15}{10x}$$

(1)

$$\frac{3}{5} \xrightarrow{\times 2x} \frac{(2x)\,3}{(2x)\,5} = \frac{6x}{10x}$$

Add the fractions:

$$\frac{20x+15}{10x} + \frac{6x}{10x} = \frac{20x+15+6x}{10x}$$

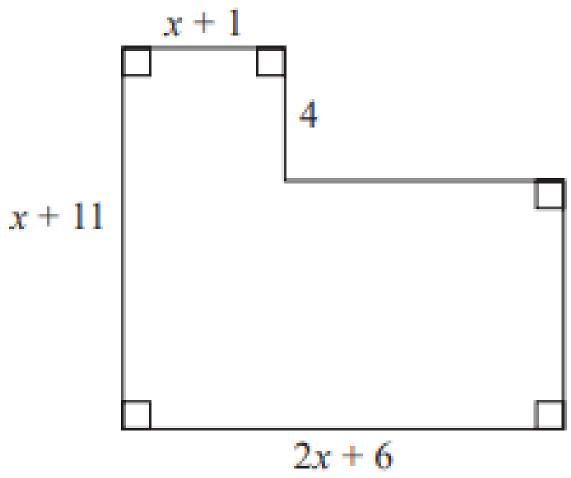
(1)

$$\boxed{= \frac{26x+15}{10x}}$$

(1)

QUESTION

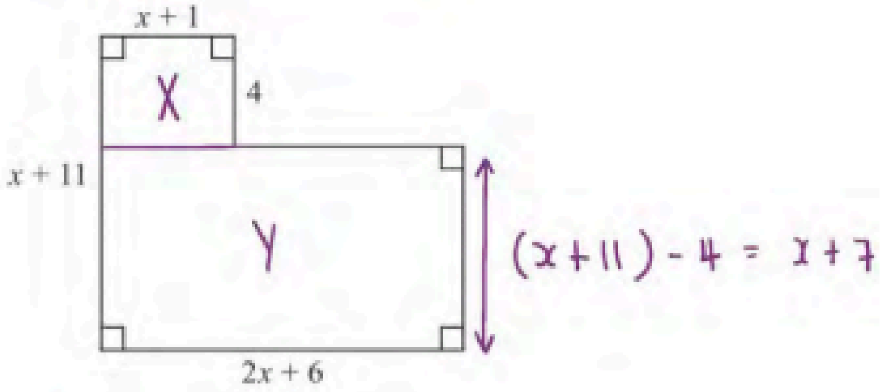
Here is a shape with all its measurements in centimetres.



The area of the shape is A cm^2

Show that $A = 2x^2 + 24x + 46$

MODEL ANSWER



Area of X :

$4(x+1) = 4x + 4$

Area of Y : (1)

$(2x+6)(x+7)$

$= 2x^2 + 14x + 6x + 42$

$= 2x^2 + 20x + 42$

Total area of shape : (1)

$(4x + 4) + (2x^2 + 20x + 42)$

$= 2x^2 + 24x + 46$ (1)

(Total for Question 14 is 3 marks)

QUESTION

Ted is trying to change $0.\dot{4}\dot{3}$ to a fraction.

Here is the start of his method.

$$x = 0.\dot{4}\dot{3}$$

$$10x = 4.\dot{3}\dot{4}$$

$$10x - x = 4.\dot{3}\dot{4} - 0.\dot{4}\dot{3}$$

Evaluate Ted's method so far.

MODEL ANSWER

Educate Cloud

Here is the start of his method.

$x = 0.\dot{4}\dot{3}$ $\quad x = 0.434343...$

$10x = 4.\dot{3}\dot{4}$ $\quad 10x = 4.343434...$

$10x - x = 4.\dot{3}\dot{4} - 0.\dot{4}\dot{3}$

Evaluate Ted's method so far. (1)

Ted does not eliminate the recurring decimals.

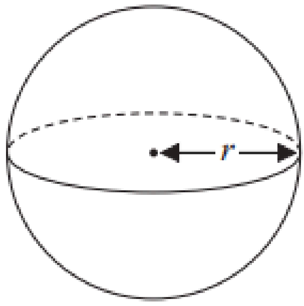
NOV 2022

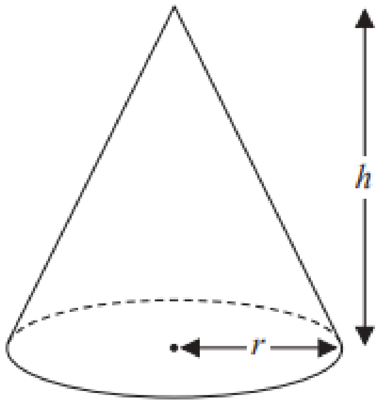
QP and MA

HIGHER

QUESTION

Here is a solid sphere and a solid cone.





Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of cone = $\frac{1}{3}\pi r^2 h$

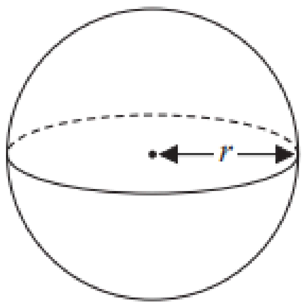
All measurements are in cm.

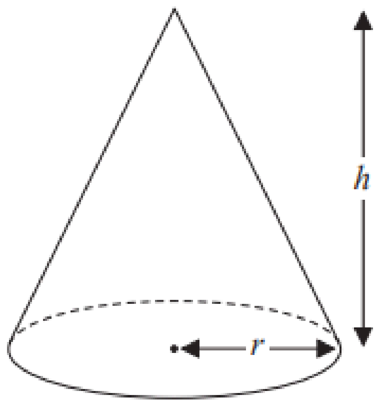
The volume of the sphere is equal to the volume of the cone.

(a) Find $r : h$

Give your answer in its simplest form.

Here is a solid sphere and a solid cone.





Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of cone = $\frac{1}{3}\pi r^2 h$

All measurements are in cm.

The volume of the sphere is equal to the volume of the cone.

(a) Find $r : h$

Give your answer in its simplest form.

MODEL ANSWER

(a) Find $r:h$
Give your answer in its simplest form.

volumes equal:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h \quad (1)$$

$\downarrow \times 3$

$$4\pi r^3 = \pi r^2 h$$

$\downarrow \div \pi$

$$4r^3 = r^2 h$$

$\downarrow \div r^2$

$$4r = h$$

$r : h$

$1 : 4$ (1)

h is four times r
so h is bigger

1:4

(2)

(b) Find $r:h$

Give your answer in the form $1 : \sqrt{n}$ where n is an integer.

surface area = SA

SA of sphere = $4\pi r^2$

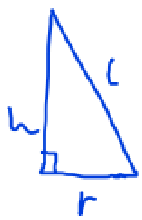
SA of cone = $\pi r l + \pi r^2$

$4\pi r^2 = \pi r l + \pi r^2$ (1)

$4r = l + r$

$l = 3r$

from right angled triangle:



$h = \sqrt{l^2 - r^2}$

sub in $l = 3r$

$h = \sqrt{(3r)^2 - r^2}$ (1)

$h = \sqrt{8r^2}$

$h = \sqrt{8}\, r$ (1)

$r : h$

$1 : \sqrt{8}$ (1)

QUESTION

Educate Cloud

Here are the first five terms of a geometric sequence.

$$\sqrt{5} \qquad 10 \qquad 20\sqrt{5} \qquad 200 \qquad 400\sqrt{5}$$

(a) Work out the next term of the sequence.

The 4th term of a different geometric sequence is $\frac{5\sqrt{2}}{4}$

The 6th term of this sequence is $\frac{5\sqrt{2}}{8}$

Given that the terms of this sequence are all positive,

(b) work out the first term of this sequence.
You must show all your working.

MODEL ANSWER

Educate Cloud

(a) Work out the next term of the sequence.

finding the common ratio:

$r = \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{10}{5}\sqrt{5} = 2\sqrt{5}$ ①

so next term is

$400\sqrt{5} \times 2\sqrt{5}$

$= 400 \times 2 \times 5$

$= 400 \times 10$

$= 4000$ ①

4000

The 4th term of a different geometric sequence is $\frac{5\sqrt{2}}{4}$

The 6th term of this sequence is $\frac{5\sqrt{2}}{8}$

Given that the terms of this sequence are all positive,

(b) work out the first term of this sequence.
You must show all your working.

If first term is a, 4th term is ar^3 so we can do 4th term $\div r^3$ to get a.

let the common ratio be r.

$\frac{5\sqrt{2}}{4} \times r \times r = \frac{5\sqrt{2}}{8}$

$r^2 = \frac{5\sqrt{2}}{8} \div \frac{5\sqrt{2}}{4}$

$= \frac{5\sqrt{2}}{8} \times \frac{4}{5\sqrt{2}}$

$r^2 = \frac{1}{2}$ ①

$r^2 = \frac{1}{2}$

$r = \pm\frac{1}{\sqrt{2}}$

$r = \frac{1}{\sqrt{2}}$

ignore $r = -\frac{1}{\sqrt{2}}$ as all terms are positive

first term is

$\frac{5\sqrt{2}}{4} \div \left[\frac{1}{\sqrt{2}}\right]^3$ ①

$= \frac{5\sqrt{2}}{4} \times 2\sqrt{2}$

$= \frac{10}{4} \times 2$

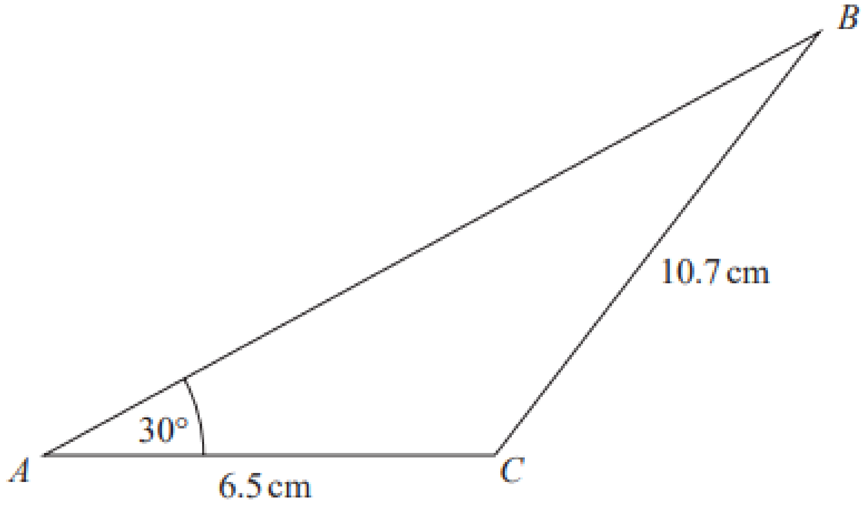
$= 5$ ①

5

(3)

QUESTION

Here is a triangle ABC.



Work out the value of $\sin ABC$

Give your answer in the form $\frac{m}{n}$ where m and n are integers.

MODEL ANSWER

using sine rule:

$$\frac{\sin ABC}{6.5} = \frac{\sin 30}{10.7} \quad (1)$$

$$\sin ABC = \frac{\frac{1}{2} \times 6.5}{10.7} \quad (1)$$

as $\sin 30 = \frac{1}{2}$ (1)

$$10.7 = \frac{107}{10}$$

$$6.5 = \frac{13}{2}$$

multiplying by reciprocal

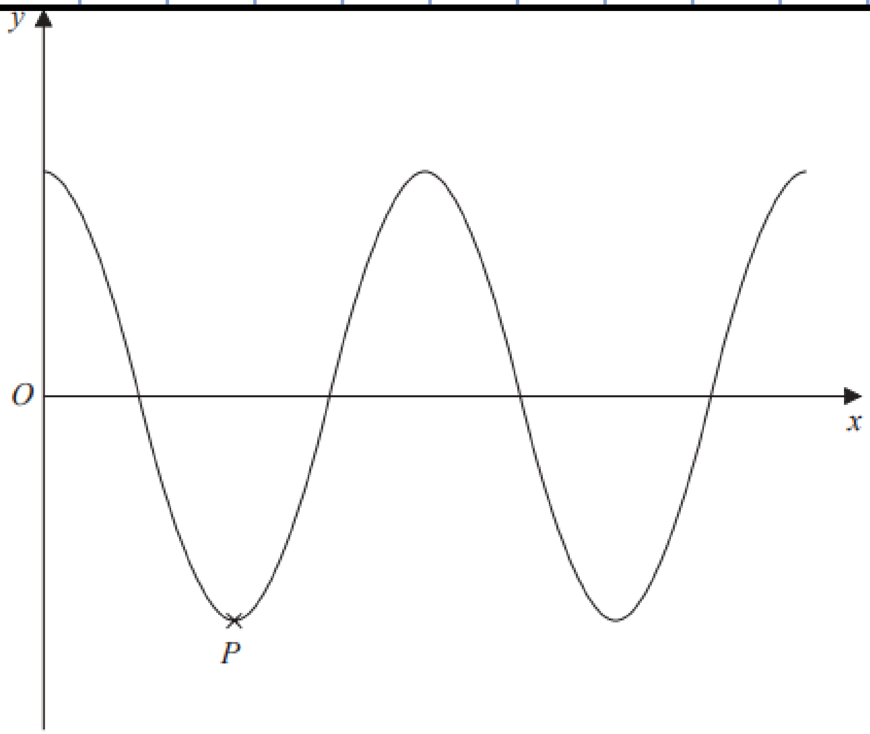
$$\sin ABC = \frac{1}{2} \times \frac{13}{2} \times \frac{10}{107}$$

$$= \frac{1}{2} \times 13 \times \frac{5}{107}$$

$$= \frac{65}{214} \quad (1)$$

$\frac{65}{214}$

QUESTION



The diagram shows a sketch of part of the curve with equation $y = \cos x°$
P is a minimum point on the curve.

Write down the coordinates of P.

MODEL ANSWER

Educate Cloud

Write down the coordinates of P.

The first minimum point on $y = \cos x$ is $(180, -1)$.

(180 ① , −1 ①)

QUESTION

Alfie has 11 cards.

He has

3 blue cards
7 green cards
and 1 white card.

Alfie takes at random 2 of these cards.

Work out the probability that he takes cards of different colours.

MODEL ANSWER

Educate Cloud

Alfie can either take cards of different colours or take cards of the same colour.

Hence P(takes different colours) = 1 − P(takes same colour) since probabilities add to 1.

$P(\text{takes two blue cards}) = \frac{3}{11} \times \frac{2}{10} = \frac{6}{110}$

$P(\text{takes two green cards}) = \frac{7}{11} \times \frac{6}{10} = \frac{42}{110}$

$P(\text{takes two white cards}) = \frac{1}{11} \times \frac{0}{10} = 0$ (1)

Add all together:

$P(\text{takes same colour}) = \frac{6}{110} + \frac{42}{110} + 0 = \frac{48}{110}$ (1)

$P(\text{takes different colours}) = 1 - \frac{48}{110} = \frac{62}{110}$ (1)

QUESTION

Solve $\frac{1}{x} - \frac{1}{x+1} = 4$

Give your answer in the form $a \pm b\sqrt{2}$ where a and b are fractions.

MODEL ANSWER

Educate Cloud

Give your answer in the form $a \pm b\sqrt{2}$ where a and b are fractions.

$$\frac{1}{x} - \frac{1}{x+1} = 4$$

$$\frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} = 4$$

$$\frac{x+1-x}{x(x+1)} = 4 \quad (1)$$

$$\frac{1}{x(x+1)} = 4$$

$$1 = 4x(x+1)$$

$$1 = 4x^2 + 4x$$

$$4x^2 + 4x - 1 = 0 \quad (1)$$

$$4x^2 + 4x - 1 = 0$$

quadratic formula:

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times -1}}{2 \times 4} \quad (1)$$

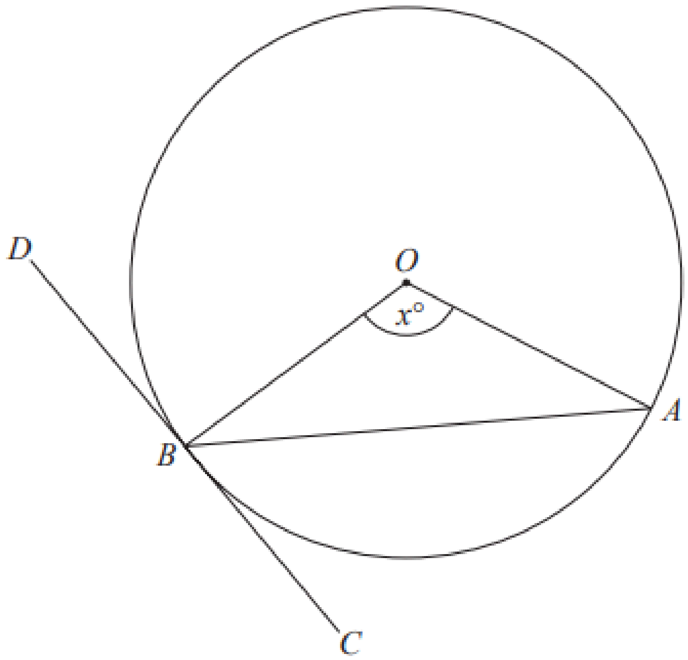
$$x = \frac{-4 \pm \sqrt{32}}{8}$$

$$\sqrt{32} = \sqrt{2 \times 16} = \sqrt{2} \times \sqrt{16} = 4\sqrt{2}$$

$$x = \frac{-4 \pm 4\sqrt{2}}{8} \quad (1)$$

$$x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{2} \quad (1)$$

QUESTION



A and *B* are points on a circle, centre *O*.
DBC is the tangent to the circle at *B*.
Angle $AOB = x°$

Show that angle $ABC = \frac{1}{2}x°$

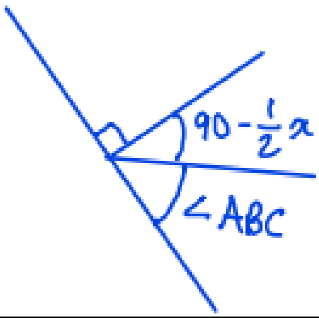
You must give a reason for each stage of your working.

MODEL ANSWER

$\angle OBD = 90°$ (1) the tangent to a circle is perpendicular to the radius

$\triangle OAB$ is isosceles so base angles $= \dfrac{180 - x}{2} = 90 - \frac{1}{2}x$ (1)

DBC is a straight line so angles add up to 180°



$$90 + 90 - \frac{1}{2}x + \angle ABC = 180$$

$$-\frac{1}{2}x + \angle ABC = 0$$

$$\angle ABC = \frac{1}{2}x$$ (1)

QUESTION

Work out the value of $\left(\frac{8}{27}\right)^{\frac{4}{3}}$

MODEL ANSWER

Educate Cloud

$$\left(\frac{8}{27}\right)^{4/3} = \left(\frac{8^{1/3}}{27^{1/3}}\right)^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

(1) (1)

$\frac{16}{81}$

QUESTION

Educate Cloud

(a) Prove that

$$(2m + 1)^2 - (2n - 1)^2 = 4(m + n)(m - n + 1)$$

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4

(b) Is Sophia correct?
You must give reasons for your answer.

MODEL ANSWER

Educate Cloud

$(2m + 1)^2 - (2n - 1)^2 = 4(m + n)(m - n + 1)$

LHS: $(2m+1)^2 - (2n-1)^2$ } difference of two squares

$(2m+1+[2n-1])(2m+1-[2n-1])$ (1)

$= (2m+2n)(2m-2n+2)$ (1) } factor out 2 from both brackets

$= 2(m+n) \times 2(m-n+1)$

$= 4(m+n)(m-n+1)$

= RHS (1)

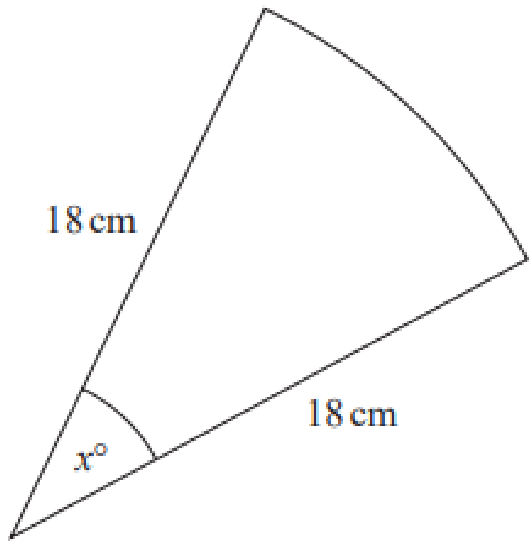
(b) Is Sophia correct?
You must give reasons for your answer.

Yes, because 2m+1 and 2n-1 are both odd numbers and the right-hand side is divisible by 4. (1)

(1)

QUESTION

The diagram shows a sector of a circle of radius 18 cm.



The length of the arc is 4π cm.

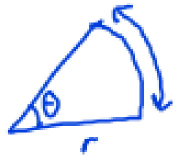
Work out the value of x.

MODEL ANSWER

The length of the arc is 4π cm.

Work out the value of x.

arc length formula $= \frac{\theta}{360^\circ} \times 2\pi r$



we have

$\theta = x$

$r = 18$

$\frac{x}{360} \times 2 \times \pi \times 18 = 4\pi$ (1)

$\frac{x\pi}{10} = 4\pi$

$\div \pi$

$\frac{x}{10} = 4$

$\times 10$

$x = 40$ (2)

$x =$ 40